

THE EFFECT OF INSTRUCTION IN SPATIAL
VISUALIZATION ON SPATIAL ABILITIES AND
MATHEMATICAL PROBLEM SOLVING

By

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This study investigated how instruction in spatial visualization affected a student's ability levels for spatial visualization and mathematical problem solving. The study probed the nature of spatial visualization and examined its correlation to problem solving performance.

Three aptitude tests, Card Rotation, Cube Comparison, and Punched Holes, were selected to measure spatial visualization. A problem solving inventory was used in addition to the spatial tests to determine the relationship between problem solving and spatial visualization. The problems were equally divided among three categories: spatial problems, analytical problems, and problems equally spatial and analytical.

The ten week instructional program was administered to 102 sixth grade students. The first and last weeks were

reserved for testing students on the spatial battery and the problem solving inventory. In the intervening eight weeks, the experimental group devoted one 45 minute period per week to developing spatial skills. Students manipulated three dimensional models, imagined the movement of those models, practiced transformations with two dimensional drawings, and experienced some problem solving activities.

The results of the study produced three major conclusions. First, spatial visualization is composed of at least two component parts. Even though all three spatial tests appeared to reflect the definition of spatial visualization, the Punched Holes test was not significantly correlated to the other two spatial tests and appeared to be measuring a different skill. Second, spatial visualization is a good predictor of general problem solving. Surprisingly, the strongest correlation was with the analytic problems subset, while the spatial problems subset was not significantly correlated to spatial visualization. One possible explanation for this result is that high spatial skills compensate for some low analytic skills, while low spatial skills are not as easily compensated for by analytic skills. Third, spatial visualization is a trainable attribute. Students in the experimental classes made significant gains in spatial scores. There was not a corresponding gain in problem solving scores, however. There were more gains in the analytic subset than the spatial subset, but the change for the whole inventory was not significant.

CHAPTER I STATEMENT OF THE PROBLEM

Introduction

The Second National Assessment of Educational Progress, completed in 1978, showed that while 85 percent of the 17 year old students tested could solve arithmetic exercises involving the four basic operations with whole numbers, only 25 percent could solve similar exercises embedded in story problems (Carpenter, Corbitt, Kepner, Linquest, and Reys, 1980). Work as early as 1925 indicated mathematics educators and researchers were aware of the need to improve the instructional strategies for teaching problem solving. Conflicting results have led more recent researchers such as Gorman (1963) and Kilpatrick (1971) to review studies in an attempt to identify which abilities contribute to effective problem solving. This study examines one of the aptitudes which may be necessary for successful problem solving.

Statement of the Research Problem

This study investigated how instruction in spatial visualization affected a student's ability levels for spatial visualization and mathematical problem solving. To help determine the effects, the study probed the nature of the aptitude called spatial visualization and its correlation to problem solving performance.

Background

Teaching students to solve work problems has remained one of the most difficult assignments facing mathematics educators. Considerable research has been generated in an attempt to determine strategies that will teach successful problem solving. Studies have tended to explore one of two strategies. The first strategy is to teach problem solving by teaching students to translate the verbal statement phrase by phrase into a mathematical statement. The second strategy is to teach problem solving by teaching students to utilize a set of heuristics.

Researchers employing the translation method have reported mixed results. Studies, as typified by Dahmus (1970), have tended to be successful when the problem content involved arithmetic or simple algebra solutions. The method is notably less successful when the problem requires multiple steps or a higher level of thought process in the solution. The level of transfer from one type of problem to another is also low for the translation method.

Researchers who favor problem solving through the use of heuristics have also reported mixed results. In an attempt to achieve more consistent results, researchers began to analyze the abilities needed to successfully use this strategy. One group of researchers has attributed the variance in problem solving performance to a general intelligence factor, computational skills, and reading ability. Another group of researchers has attempted to show that

other abilities also account for part of the variance. As early as 1935 some mathematicians felt that mathematical ability was a composite of general intelligence and the ability to visualize number and space configurations and to retain those configurations as mental pictures (Brinkmann, 1966).

Outline of the Study

This study examined the nature of spatial visualization, its relationship to mathematical problem solving, and the effects of instruction on the aptitude and the relationship. Students were tested at the beginning of the project to determine an initial level of spatial visualization and problem solving skills. After an instructional program, students were again tested on the same skills to establish the effects of the program on the skills.

The study was conducted in the Lowndes County Public School System in Lowndes County, Georgia. Sixth grade students at two elementary schools were selected to be the sample population. Students at the Lake Park Elementary School were the control group while students at Parker Mathis Elementary School were the experimental group and received the instructional program in visualization skills.

The instructional phase of the study lasted ten weeks. The first and last weeks were devoted to pre and post testing. Students in the experimental group received eight weeks of spatial training. The training consisted of physically manipulating three dimensional objects, mentally

manipulating two and three dimensional objects, and applying these skills to problem solving situations. Students in the control group followed the normal mathematics curriculum.

Three instruments were selected to measure spatial visualization. These tests had been identified as measures of the ability by several previous studies. The three measures were the Punched Holes Test, the Card Rotations Test, and the Cube Comparison Test. The scores of the three tests were combined to form a single measure of spatial visualization called the spatial index.

A problem solving inventory was used in addition to the spatial battery to examine the relationship between mathematical problem solving and spatial visualization. A validated problem solving inventory, appropriate for the sixth grade students chosen to be the subjects, did not exist in the literature describing previous research in problem solving. The inventory used in the study was synthesized from the problem sets found in other research studies.

The study explored three major areas. The first area was the nature of the aptitude called spatial visualization. To explore this aptitude, the study attempted to answer the question:

1. Does the aptitude called spatial visualization consist of a single ability or does it have two or more component parts?

The second area explored was the relationship between spatial visualization and mathematical problem solving. To discover what relationship exists, the study attempted to answer the following questions:

1. Is there a relationship between a students' spatial index and their mathematical problem solving performance?
2. Is there a relationship between a students' spatial index and their problem solving performance on spatial problems?
3. Is there a relationship between a students' spatial index and their problem solving performance on analytic problems?

The third area investigated was the effect of an instructional program on spatial visualization and the relationship between spatial visualization and mathematical problem solving. To determine the effects, the study attempted to answer the following six questions:

1. What effect will instruction in visualization skills have on a students' spatial visualization aptitude?
2. What effect will instruction in visualization skills have on problem solving performance?
3. What effect will instruction in visualization skills have on the problem solving performance on spatial problems?
4. What effect will instruction in visualization skills have on the problem solving performance on analytic problems?

5. Will instruction in visualization skills affect the problem solving performance of high spatial students differently than that of low spatial students?
6. Will instruction in visualization skills affect problem solving performance on spatial problems differently than that on analytic problems?

Statement of Hypotheses

The above questions generated the following set of hypotheses on the nature of spatial visualization and its relationship to problem solving:

1. There is not an indivisible aptitude which is called spatial visualization.
2. There is no correlation between spatial visualization and mathematical problem solving performance.
3. There is no correlation between spatial visualization and problem solving performance on spatial problems.
4. There is no correlation between spatial visualization and problem solving performance on analytic problems.
5. There is no significant difference in spatial visualization ability between the control and experimental groups after the experimental group receives instruction in visualization skills.
6. There is no significant difference in problem solving performance between the control and experimental groups after the experimental group receives instruction in visualization skills.

7. There is no significant difference between control and experimental groups in performance on spatial problems after the experimental group receives instruction in visualization skills.
8. There is no significant difference between control and experimental groups in performance on analytic problems after the experimental group receives instruction in visualization skills.
9. There is no significant difference between high spatial students and low spatial students in problem solving performance after the experimental group receives instruction in visualization skills.
10. There is no significant difference between the experimental group's performance on spatial problems and their performance on analytic problems after they receive instruction in visualization skills.

Definition of Terms

In reviewing research studies on spatial ability and problem solving, it became apparent that different researchers were using the same term to denote different concepts. Thus it seemed appropriate to compile a list of such terms and give the definition used in this study.

- a. Spatial Visualization is the ability to recognize the relationship between parts of a given visual configuration and the ability to mentally manipulate one or more of those parts. Students with good

spatial visualization skills will be able to formulate a mental image from a verbal description and will be able to rotate that image to different perspectives.

- b. Spatial Index is the measure of each subject's spatial visualization ability. In this study it is the sum of the Z scores on the three tests in the spatial battery.
- c. A Problem is a situation which requires resolution but for which a procedure to determine the outcome is not immediately clear. The amount of knowledge and experience a person brings to the situation determines if it qualifies as a problem. A situation which qualifies as a problem for some students would merely be an exercise for others.
- d. Spatial Problems are situations which qualify as problems and which may be resolved most easily and efficiently through a visual process. Students would typically sketch or mentally imagine a picture to facilitate achieving the solution. This does not preclude the possibility of some students using an analytical solution process.
- e. Analytic Problems are situations which qualify as problems and which can be resolved most efficiently through an abstract or symbolic process. Students would typically write an equation or number sentences to facilitate achieving the solution. This

does not preclude the use of visual aids by some students during resolution of the problem.

- f. Problem Solving Performance is the measure of each subject's ability to solve problems. In this study it is the number of items solved correctly on the problem solving inventory.

Limitations

The research for this paper was conducted within the bounds of several limitations. The sample size was limited to the sixth grade students at two elementary schools. To increase the sample size, the instructional program would have had to be conducted in a third elementary school. This would have required the use of another instructor, and would have introduced variability due to differences in instructors.

A second limitation was the requirement to maintain intact classes. This prevented the random assignment to students of the control group or the experimental group. This reduced the degrees of freedom available for statistical analysis, but it also eliminated some threats to external validity.

The third limitation involved the amount of time available to students to solve the problem solving inventory. The amount of time limited the number and variety of problems that could be chosen for the inventory. A more comprehensive inventory might have provided a more accurate picture of each student's ability.

Organization of Paper

This paper is divided into five chapters. Chapter II reviews previous research that dealt with problem solving using visual aids, the nature of spatial visualization, and the effects of spatial instruction. Chapter III describes the research design and the method of the implementation. Chapter IV contains an analysis of the data collected during the study. Chapter V summarizes the results and offers some conclusions and implications for further research.

CHAPTER II REVIEW OF RESEARCH

Overview

This chapter reviews research pertaining to the investigation of the nature of spatial visualization, the relationship between spatial visualization and problem solving, and the effects of instruction on both of these. The review is divided into three sections. The first section is devoted to research studies which contribute to the understanding of the relationship between problem solving and spatial visualization. The conflicting results produced by this body of research set the stage for the second section which examines research that attempted to define spatial visualization. The final section of this chapter reviews what other researchers have observed about the effect of instruction on a student's spatial visualization skill level.

Spatial Visualization and Problem Solving

Gorman (1968) reviewed 293 research studies dealing with solving word problems. He found the results inconclusive or conflicted with results from other studies. Studies which have examined the facilitative effect of including visual aids in instruction have also produced differing results. Runquist and Hunt (1961) compared a verbal presentation to a pictorial presentation in a concept learning task. They found that performance was significantly better when the

stimulus was verbal rather than pictorial. The researchers suggested that this may in part be due to the fact that during the performance of the task, subjects had to respond verbally and performance was better when the stimulus was in the same medium as the required response.

Kulm, Lewis, Omari and Cook (1974) used ninth grade algebra students as subjects in comparing five different treatments of word problems, three of which contained pictorial aids while two were strictly verbal. They found that the verbal versions were significantly superior for low IQ groups. Middle and high IQ groups performed better on the strictly verbal versions than on those which incorporated pictures, but the scores were not significantly better. Kulm et al. suggested that the presence of a picture may have interfered with problem solving clues, especially for low IQ subjects.

Sherrill (1972), in a similar study, reported opposite results. Sherrill used tenth grade geometry students to examine the effect of providing accurate sketches, inaccurate sketches, or no sketches with word problems on the students' ability to solve the problem. He found that having an accurate sketch was superior to having no sketch, but that having no sketch was superior to having an inaccurate sketch. Students in high and middle IQ groups benefited more from accurate sketches than students in low IQ groups.

Bassler, Beers, and Richardson (1975) compared two strategies for teaching ninth grade algebra students how to find the solution to verbal problems.

One strategy encouraged students to develop their own pictures as a part of the solution process. This strategy produced significantly higher scores on the Problem Solution Test. Pictures were not required, however, and the authors reported no measure of the quantity of pictures used by students.

These four studies have two major flaws. The first problem is that, with the exception of one of the five treatments of Kulm et al. subjects could have worked on the tasks without attending to the visual aids. Koenke (1970) found not only did pictures not help in recalling descriptive paragraphs, but specific directions to attend to the pictures were not effective. The four cited studies did not include any measure to indicate how much or in what way the visual aids were used by the subjects.

The second problem concerns the subjects ability to use the visual aids. None of the studies measured the spatial visualization ability level of their subjects, although it is a key factor in the students' ability to use a sketch to their advantage. The opposite results of the similar studies would seem to indicate that some aspect critical to the use of pictorial aids in problem solving was overlooked.

Further evidence to support the need for a closer look at how students use visualization comes from the opposite conclusions reached by Paige and Simon (1966) and Krutetskii (1971) in similar studies. Paige and Simon had students think aloud as they solved word problems. They found that

students developed a mental picture of the situation given in the problem. When students were given contradictory problems, which contained impossible physical situations, Paige and Simon concluded that good problem solvers were more likely to "see" the contradiction than poor problem solvers. Krutetskii found that less capable students tended to use a concrete model of the problem and thus were more likely to discover contradictions. He suggests that more capable secondary students are more proficient at identifying types of problems and solving problems within types by set methods. Thus the more capable students would not envision the physical situation and would not notice that the information presented a contradiction.

Several more recent studies directly measured spatial visualization aptitudes and attempted to see what effect varying skill levels had on utilizing instruction and solving problems. Frandsen and Holder (1969) found that the ability to find successful solutions to complex problems was related to spatial visualization aptitude. Students with high ability levels did significantly better solving complex problems on a pretest than students with low ability level. After an instructional program which taught techniques for representing data and conditions, students were tested again. Those who began the study with high spatial visualization scores did not show a significant change in problem solving. However, students who had low spatial visualization scores showed a significant gain in problem

solving. Thus it would appear the spatial visualization aptitude interacted with the training.

A series of studies utilizing the aptitude-treatment interaction model examined how spatial visualization ability levels affected student performance with verbal and pictorial presentations. Carry (1968) used spatial visualization, verbal, and general reasoning aptitudes as predictors of student success in answering questions after a primarily pictorial presentation or a strictly verbal, analytical presentation. He found the level of a student's spatial visualization aptitude was the best predictor of student success within the verbal treatment, while verbal aptitude was the best predictor of student success in the pictorial treatment. Hamilton (1969) used the same three aptitudes and produces parallel results, even though the subject matter was quite different. In both studies the results were the opposite of what the researchers predicted.

Webb and Carry (1975) did a follow up study to Carry's original work. Webb and Carry used the theoretical model developed by Melton (1967) to construct their hypotheses, revise the instructional treatments, and select aptitude tests. The analysis using Melton's model suggested that the result obtained by Carry and Hamilton did conform to theoretical expectations. The model predicted high aptitude levels would compensate for the weak areas of the treatments. Webb and Carry, however, failed to obtain any significant interactions between the treatments and aptitudes.

Eastman and Carry (1975) continued the investigation of this research problem. Eastman and Carry attributed the difference between results obtained by Carry and by Webb and Carry to a failure to match tests and treatments in inductive or deductive structuring. The tests used to measure spatial visualization were deductive in nature for both studies. Carry's original treatments were deductive, while those revised by Webb and Carry treatments were inductive. Thus the spatial tests should have been a better predictor for Carry's treatment. Eastman and Carry again revised the treatments to make them more deductive and repeated the study. Their results supported Carry's original finding, showing general reasoning a significant predictor for the verbal treatment but not for the graphical treatment, while spatial visualization was a significant predictor for both treatments.

Behr and Eastman (1975) did a further follow up to the studies. Although the treatments reflected the inductive-deductive variable and increased the number and kind of tests used to measure aptitude, the study failed to produce either a significant difference between the pictorial and verbal treatments or a correlation between aptitude and the criterion measure that was significantly different across groups. The authors suggest that one of the main obstacles in identifying viable treatments and aptitude variables that consistently produce attitude-treatment interactions is the lack of a theoretical background concerning the structure of the intellect. They further suggest more

research is needed to identify and measure aptitudes necessary for mathematical learning.

Moses (1979) attempted to determine if spatial visualization was a necessary aptitude for problem solving. She conducted a series of lessons designed to encourage spatial thinking. Students who received spatial instruction improved significantly in their problem solving performance. Students with low reasoning ability profited more from the instruction than students with high reasoning ability.

The Project Talent survey results discussed by Flanagan (1964) further support a link between spatial visualization and problem solving. Results from several types of mathematics tests were correlated with various aptitude measures. Results of the spatial tests correlated highly with results from the mathematical tests. The NLSMA data (Wilson and Begle, 1972) also showed a high correlation between mathematics tests and spatial tests.

Lean and Clements (1981) did not find a correlation between spatial ability and mathematical performance. Their battery of spatial tests included ones similar to those used in the aptitude-treatment-interaction series and by Moses. Their results, however, showed that spatial ability and knowledge of spatial conventions had little influence on scores for pure mathematical and applied mathematics tests. Lean and Clements conjectured that their finding differed from earlier results because earlier studies used unfamiliar and nonroutine problems to measure problem solving abilities while they used routine work problems.

Battista, Wheatley and Talsma (1982) also failed to find a link between spatial visualization and mathematical performance. Their study was an aptitude-treatment-interaction study that hypothesized that high spatial students would perform differently in verbal and verbal spatial treatments of algebraic structures than low spatial students. The data did not show a positive correlation between the measure of spatial visualization. Neither did it show a significant interaction between the treatments and the aptitude. Battista et al. speculated that this lack of expected results may imply that not all areas in mathematics readily utilize spatial abilities. Topics such as algebraic structures may be processed in a verbal symbolic mode and thus require little spatial visualization.

The research reviewed in this section presented conflicting results. The contradictions can be largely attributed to differing interpretations of spatial visualization and the uneven quality of problem solving tests. Research needs to clarify a definition of spatial visualization and determine which existing tests, if any, measure the aptitude as per that definition. In a similar vein, Glaeser (1983), in a review of problem solving research, found little agreement on what constitutes a problem. Further, he found little evidence that researchers were even aware of the different definitions they used in their studies. A standard definition of problem solving must be accepted by researchers, and set of appropriate problems identified.

Spatial Visualization

Spatial visualization abilities have been a part of the study of aptitudes since Galton included a study of imagery as part of his systematic psychological studies in 1883. They have been of interest to mathematicians for almost as long. In 1935 the Australian mathematician H.R. Hamley stated that mathematical ability was composed of general intelligence, visual imagery, the ability to perceive numbers, and the ability to perceive space configurations and retain them as visual images (McGee, 1979). In May, 1980, representatives of the International Commission on Mathematics Instruction and the World Confederation of Organizations of the Teaching Profession stated that one of the goals of mathematics education should be to develop spatial perceptions while working with two and three dimensional models (Morris, 1981). Before it is possible to establish what part spatial visualization plays in mathematics and problem solving, it is necessary to establish what spatial visualization is.

The advent and refinement of factor analysis as a statistical technique assisted researchers in this area. Thorndike (1921) identified spatial visualization as a major and relatively independent component of intelligence. Thurstone (1938) also reported it to be an important factor, independent of his General Intelligence factor. In fact, Fruchter (1954), summarizing factorial results to that time, found that a spatial factor was the second most frequently identified factor, following only the verbal factor.

Although researchers have consistently found an independent spatial factor when testing student abilities, they have not been consistent in their approach towards analyzing the ability. Smith (1964) has summarized the different views adopted by British and American researchers. The British view the realm of abilities as a continuum, with verbal and numerical ability at one end and spatial and mechanical ability at the other. Americans tend to view general ability as a composite of a large number of independent and equally important abilities.

This lack of conformity in approach has been one factor contributing to the controversy concerning the precise nature of spatial visualization. Thurstone's original analysis of primary mental abilities done in 1938 included only one spatial factor. He defined this spatial factor as a facility with spatial and visual imagery. This was one of the few studies, however, which limited spatial visualization to one component. In a later study, Thurstone (1950) reported that his spatial visualization factor was in fact composed of two independent factors, which he labeled spatial relations and visualization. Spatial relations was defined as the ability to recognize a rigid configuration when it is viewed from different angles. Visualization was defined as the ability to envision a configuration with movement among the internal pieces of the configuration.

French (1951) also concluded that there were two spatial factors. He termed his two factors spatial orientation and

spatial visualization. Spatial orientation, as defined by French, is the ability to remain unconfused when the orientation of a configuration is changed. Spatial visualization is the ability to comprehend the imaginary movements of three dimensional objects in space.

Guilford, Fructer, and Zimmerman (1952) found two spatial factors when they analyzed the Army Air Forces Sheppard Field Battery results. They, too, labeled their two factors spatial orientation and spatial visualization but defined them differently from French. They defined spatial orientation as the ability to appreciate spatial relationships with reference to the body of the observer. Spatial visualization was defined as the ability to imagine movements and transformations in visual objects.

Michael, Zimmerman, and Guilford (1950) tested for differences in psychological properties of spatial relations and visualization. These differences were manifested in the content of two types of tasks the two factors would facilitate, and in the operational procedures subjects would use to carry out the tasks. They hypothesized spatial relations to be "the ability to comprehend the arrangement of elements within a visual stimulus pattern, primarily with reference to the human body" (p. 190). Visualization was hypothesized to be "the ability that requires the mental manipulation of visual images" (p. 190). This study found that there were differences between the two factors. Follow up interviews with subjects showed, however, that, at least on more

challenging problems, one ability was not used exclusively. Subjects would use one ability to attempt the solution and the other ability to support or verify that solution.

Michael, Guilford, Fruchter, and Zimmerman (1957), in a further attempt to separate spatial factors, identified three factors. The first two represent a refinement of the factors in the study by Guilford et al. (1952). Spatial relations and orientation was defined as the ability to understand the relationship among elements in a given stimulus pattern with respect to the observer's body. Visualization was the ability to mentally manipulate objects through a specific sequence of movements. The third factor introduced was termed kinesthetic imagery. This was defined to be discrimination of left and right with respect to the location of the body of the observer. This last factor did not have as much supporting data as the first two factors, but has also been identified in later work by Thurstone (1950).

Other studies have rejected the hypothesis of two independent spatial factors. French (1965) expected two factors, which he termed Space and Visualization, to be separate factors in his factor analysis results. That did not happen. French further showed that this was not just a defect of the rotation during analysis. Moses (1979) also rejected a hypothesis that spatial visualization had more than one component.

Guttman and Shoham (1982) used Smallest Space Analysis to examine eight spatial tests. Their results showed that

three facets, rule-task, dimensionality and rotation, all formed distinct regions. They speculated that there may be other facets that contribute to the structure of spatial abilities.

In an article reviewing the history of measurement of spatial abilities, Fruchter (1954) cited three main problems with spatial research results. The first problem was that research studies have produced different results as to the number and nature of spatial factors. The second was that these differences have led researchers to formulate varying definitions for spatial factors. The third problem was that spatial tests do not load consistently on the same factors. Fruchter suggested that these problems were due to differences in populations and in the composition of the test battery. Even researchers with similar definitions for spatial factors have at times chosen different sets of tests to measure one defined ability.

Thurstone (1938) developed some of the most commonly used spatial tests. Flags, Figures, and Cards and Cubes are two tests he developed to measure his spatial relations factor. In the first test, Flags, Figures, and Cards, the subject indicates whether two drawings, usually in different positions, can represent the same side of an object. In Cubes the subject decides whether two drawings, each showing three sides of a cube, can represent the same cube when the cube has a different design on each face. Two tests developed by Thurstone to measure visualization are Punched Holes and

Form Board. The Punched Holes test shows a subject how a square sheet of paper is folded several times and then has a hole punched through all the layers. The subject must then mentally unfold the paper and indicate where the holes would be on the original square. The Form Board test shows a subject several two dimensional block pieces of various shapes. The subject is required to draw lines in a larger enclosed design to show how the pieces fit within the design.

Guilford and Zimmerman (1949) developed a new test for each of their spatial factors to be used in conjunction with some of Thurstone's tests. Spatial Orientation was developed to measure their spatial relations and orientation factor. In this test the subject assesses how the position of a boat has changed from the initial picture to a second picture. Spatial Visualization was developed to measure their visualization factor. In this test the subject is shown a clock in an initial position. Verbal statements are made describing the movements of the clock. Subjects then choose from several alternatives the picture that shows the final position of the clock.

Michael, Zimmerman and Guilford (1950) used all six tests in their study that confirmed two separate spatial factors. The amount of overlap between the two factors led them to expand their study. Michael, Guilford, Fruchter and Zimmerman (1957) used the same six tests plus two more developed by Thurstone, Hands and Bolts, to identify their three spatial factors.

Several researchers have explored differences among groups as a means for explaining variance in spatial studies. The most frequently considered variable is sex. There have been many reports that females perform less well than males on spatial tests. Several researchers have suggested that this is due to a genetic basis (O'Connor, 1943; Vandenberg, 1969, 1975). Research concerning spatial abilities which measured sex differences as one aspect of the total study has produced mixed results. Bock and Kolskowski (1973), Miller (1967) and Guay (1978) all found that males did significantly better on spatial tests than did females, even when groups were matched on other abilities. Guay and McDaniel (1977) and Moses (1977) both rejected that hypothesis based on their results. Sherman (1974) reviewed the research on spatial visualization abilities and sex differences and suggested that results were due to culturization and expectations. Vandenberg (1975) suggested that the genetic trait which accounts for the earlier development of verbal ability in females may also be linked to lower spatial scores. He speculated that the early language development impedes spatial development. At this point there does not appear to be a clear answer to the sex link question.

Burnett, Lane, and Dratt (1982) examined the relationship between spatial ability and handedness. They found that individuals who showed extreme left hand or right hand preferences fell into the lowest performance group on

spatial visualization tests. Superior spatial visualization scores were made by students with mixed or weak lateralization. Thus students who did not show strong preferences for left or right handedness did better on spatial tests.

Nuttin (1965) examined the effect of socioeconomic class on spatial abilities. He showed that performance on spatial tests was less correlated to socioeconomic differences than performance on verbal tests or general intelligence tests. Bowden (1969) examined differences due to race between African and European children. He found that culture and education accounted for the differences which did exist, while race was not a significant factor.

In his review and analysis of spatial studies, Smith (1964) concluded that spatial visualization ability is positively correlated with a high level of mathematics conceptualization. That is, students who can solve high level mathematics problems generally perform better on spatial tests than those who cannot. Spatial visualization had no correlation to a student's ability to solve low level conceptualization problems that rely primarily on computation. Guay and McDaniel (1977) found a positive correlation between high achievement in mathematics and high levels of spatial visualization ability. Neither study reported a cause and effect relationship in either direction between the abilities.

Guay (1978) found that the experience level of subjects may account for a large portion of variance between groups. His research showed that subjects with much experience in activities requiring spatial thinking performed better on spatial tests than those with little experience. Miller and Miller (1977), in two controlled studies, found that performance on spatial tests improved after experience. They concluded that genetic and environmental components only determine a subject's capacity for development while functioning ability depends on experience.

Just what constitutes spatial visualization remains disputed. For the purpose of this study, the definition of spatial visualization will be a composite of the two most commonly identified spatial factors. Spatial visualization, as defined in Chapter I, will be the ability to recognize the relationship between parts of a given visual configuration and the ability to mentally manipulate one or more of those parts. The tests chosen to measure spatial visualization reflect the two abilities stated in the definition. The tests also require the abilities be utilized with both two and three dimensional representations.

Effects of Instruction

In his book on spatial visualization, Smith (1964) states that spatial abilities make an important contribution to mathematical ability that is usually ignored in public school education. McKim (1972) points out that opportunities in school for visual expression usually end early in

the primary grades. If, as Carpenter (1972) contends, the basic purpose of education is to identify and correct deficiencies in desirable skills, educators must reconsider this exclusion of spatial training.

Brinkmann (1966) instituted a study that examined programmed instruction as a technique for improving spatial visualization. While he acknowledged the lingering controversy of innate versus acquired nature of spatial skills, he attempted to demonstrate that deficiencies in perceptual skills could be overcome through learning. His results showed that functional spatial visualization skills of individuals could be improved when given appropriate training.

Moses (1979) also demonstrated that student performance on spatial tests improved after spatial visualization training. Her training program included work with two and three dimensional figures, with a primary emphasis on students actually manipulating the geometric objects. Students showed a significant gain on posttest spatial scores. In another training program emphasizing manipulation of concrete objects, Miller and Miller (1977) found that the experimental group demonstrated significant spatial visualization gains while the control group did not improve. Seven months later students had maintained that gain on test scores.

Mitchellmore (1980) lends supporting argument for a training program to improve spatial visualization. He found English students were better at three dimensional

drawing than American students. He attributed this superiority to the fact that English teachers use more manipulative at the elementary level, use diagrams more freely, and have a more informal approach to geometry than do American teachers. Mitchelmore hypothesized that the greater number of school experiences in visual perceptions accounted for the differences in drawing ability.

Carpenter's (1972) findings were less clear cut. He did find that groups that participated in one of his training programs did better than control groups. Another result concerning reliability coefficients suggested that initial rank ordering of students changed significantly on the post-test. This was supported by the fact that low I.Q. students did significantly better with programmed material when compared to low I.Q. students in control groups.

Krumboltz and Christal (1960) considered short-term effects of spatial training. They were specifically interested in how one test of spatial visualization affects the subjects performance on a second test that follows immediately. Results showed that regular forms and alternate forms of the same test were subject to practice effects. Thus even limited experience on a specific spatially oriented task improves immediate performance of that task. The practice effect was not transferable, however. Students showed no gains when an alternate type of spatial test was given.

Breslauer, Mack and Wilson (1976) examined the effects of a training program on visual perception. They contended that a deficiency in perception would impede the learning process. Both perceptual and visual development training procedures were employed. Most participants showed measurably reduced symptoms of visual inefficiency. For primary children, this increased self image and provided them with the opportunity to advance at a normal pace. Students in upper elementary levels improved their visual-perceptual skills, but were not able to transfer that to improved learning. Breslauer et al. suggested this was due to repeated failures and poor self image experienced over several years due to poor perceptual skills.

Ives and Rakow (1983) explored how language facilitated performance on spatial tasks for young children. They found that the use of language can greatly improve performance on perspective tasks, but has much less effect on rotation tasks. Spatial scores improved when language clues such as "front corner", "side", or "back" were introduced as part of perspective tasks. Perspective and rotation task performances were both improved when objects with familiar, inherent features were used. Their results showed that spatial ability was not a talent that was either present or not present in young children, but a skill that could be improved as the complexity of the problem was reduced.

Paivio (1973) has hypothesized that every task requires both verbal and spatial thought for solution. He posed three variables which he thought determined the amount of visual imagery an individual employs to solve the task. The first variable involves the number of familiar stimuli involved in the task. Tasks which involve physical objects which are familiar to the subject generate more visualization than tasks which do not involve familiar physical objects. The second variable is the extent to which directions for the task specify a visual or verbal approach to the task. Subjects are more likely to follow suggested avenues even when visual solution methods might be easier. The final variable is the processing mode used by the subject. Individual preference and amount of previous practice often influence the mode selected by different subjects.

Brinkmann (1966) stated that the first step in creating a program to improve any ability is to specify in precise behavioral language just what behavior a subject displays in achieving the desired goal. Brinkmann proposed the following four as behaviors that contribute to spatial skills: a) differentiation or discrimination, b) identification (which includes recognition and labeling), c) organization or recognizing relationships, and d) orientation. Brinkmann's training program concentrated on developing the learner's skill in discrimination and identification tasks, using both two dimensional drawings and three dimensional

manipulatives. The tasks became more difficult as the training progressed.

In his book, Experiences in Visual Thinking, McKim (1972) offers a three step program for training spatial visualization abilities: seeing, imagining, and idea sketching. McKim feels that most students have not been taught how to really see things. He acknowledges individual differences in ability levels, but feels that regardless of the inherited ability, there is a large amount of unrealized potential for visual development. McKim states:

Seeing is more than sensing: seeing requires matching an incoming sensation with a visual memory. The knowledgeable observer sees more than his less knowledgeable companion because he has a richer stock of memories with which to match incoming visual sensations. (p. 43)

Included under his topic of seeing are externalized thinking (which involves manipulating an actual object), seeing by drawing (drawing forces one to really look), pattern seeking, proportions, and learning cues for form and space. Imagining requires the use of visual recall, to mentally manipulate objects, and to examine structures and abstractions. Idea sketching allows students to test their mental picture by transferring it to paper and to bypass the reliance on verbal descriptions. McKim's training included using three dimensional models, two dimensional geometric shapes, and two dimensional representations of three dimensional objects.

Weinzweig (1978) proposed developing spatial concepts by teaching informal geometry. He relied on the Klein-Erlanger Programm, which emphasizes the transformational approach. Weinzweig felt children should begin geometry by sorting and classifying a suitable collection of solid geometric models. This would focus attention on similarities and differences and help the child formulate concepts such as straightness. The next step is helping the child learn to read pictures or drawings of three dimensional objects and be able to mentally picture the object of the picture or drawing. The final step is for the child to discover certain invariants within movement. From these concepts the child can begin to develop a geometry.

Battiste, Wheatley, and Talsma (1982) looked at a geometry course for preservice elementary teachers that included numerous spatial activities. Spatial test scores were significantly higher after the course, suggesting that the types of activities included in the course improved spatial visualization abilities. Results of multiple correlation also suggested that spatial visualization is an important factor in geometry learning.

Kilpatrick and Wirszup (1971) have collected and edited a series of Russian research studies and published them under the title of Soviet Studies in the Psychology of Learning and Teaching Mathematics. Many of these studies relate to the learner's spatial abilities. Kilpatrick and Wirszup point out in their introductions that Soviet social and

political philosophies dictate that Soviet researchers take a different view of spatial abilities than the one of innate, unchangeable ability level held by many Western psychologists. Soviet psychologists have examined how spatial abilities are influenced by instruction, and what types of instruction improve spatial abilities.

Botsmanova (1971 a and b) conducted two studies examining the role of pictorial aids in problem solving. The first of these (1971a) was similar to the first four studies reviewed in the section Spatial Visualization and Problem Solving. Students were given work problems with three types of pictorial aids: pictures showing the objects mentioned in the problem to illustrate the subject of the problem; pictures showing the objects mentioned in the problem which also show the relationships between the data; abstract spatial drawings or diagrams. The pictures showing the relationships between data were most likely to be helpful, but were not always effective. Pictures which only illustrated the subject matter, the most common type of pictorial aid found in textbooks, generally did not contribute to the solution of the problem. Abstract drawings and diagrams also did not contribute to the solution of the problem. In her second study (1971b), Botsmanova taught students to evaluate pictorial aids and to develop their own pictures showing relationships between pieces of data. These students were much more successful solving problems than students in the control group. Thus it would appear that students taught to

attune to pictorial aids and to interpret those drawings were more successful students without such training.

Chetverukhim (1971) investigated the level of development of spatial concepts in several grades. Students in first, fourth, fifth, and sixth grades were asked to imagine and then draw six common objects. Drawings by first graders showed that while they had developed spatial concepts and could perceive three dimensional ideas, they lacked the knowledge or skill to represent it in drawings. Fourth, fifth and sixth grade students demonstrated increasing abilities to represent three dimensional objects in drawings even without specific training. Older students in eighth, ninth, and tenth grades, and beginning college students were accurate drawing geometric figures in correct proportions, but had difficulty imagining and drawing plane sections of the solids. Chetverukhim attributes this difficulty primarily to the fact that textbook illustrations use only standard representations of geometric figures.

Vladimirskii (1971) cited several obstacles to the development of spatial visualization in the current curriculum. One major problem is that students are not able to distinguish between essential and nonessential features. A second major problem is the improper use of visual aids. Students are introduced to visual aids through passive viewing. Vladimirskii undertook to devise an educational strategy to counter these two problems. Preliminary exercises called for students to manipulate solids to match

drawings of the solid. Students were expected to learn to recognize a figure from a drawing of it and to orient the figure in space. After learning the conventions associated with drawings that represent three dimensional figures, students were acquainted with the conventions that imply movement in the diagram. Once the students were able to read diagrams, Vladimirskii introduced his experimental exercises. These exercises required students to 1) recognize figures in differing positions, 2) compose and decompose pictures when an element of movement is included, and 3) recognize and explain geometric relationships in concrete form. A final set of exercises was introduced to help students develop the concept of parallelism of a line and a plane.

Krutetskii (1971) described a series of problems that could be used to develop spatial visualization in a more general chapter on experimental problems. These problems demand spatial visualization as a fundamental part of the solution process. He suggested the use of both solid objects and diagrams in exploring the solution process. The content of the problems was based on informal geometry, but prior knowledge of geometric principles was not necessary and, in fact, would not help in the solution of most of the problems.

Research on the effect of instruction on spatial visualization tends to support the hypothesis that training has a positive effect on the performance level of participants.

The training programs developed by Brinkmann, McKim, Weinzwieg, Vladimirkii and Krutetskii were culled to select activities that supported the definition of spatial visualization used in this study. Those activities were modified as necessary to fit the background and development state of the sample population. The instructional program created from these activities is described in Chapter III.

CHAPTER III RESEARCH DESIGN AND IMPLEMENTATION

Overview of the Study

This study was designed to investigate spatial visualization and its relationship to problem solving. It explored three major areas:

1. What is the nature of the aptitude termed spatial visualization?
2. What is the relationship between spatial visualization and problem solving performance?
3. What is the effect of instruction in visualization skills on spatial visualization and the relationship between it and problem solving?

Chapter III describes the evaluation instruments used to measure spatial visualization and problem solving performance, the sample population of the study, the instructional program used with the experimental group, and the statistical procedures used to evaluate the results.

Selection of Evaluation Instruments

Each student in the sample population was evaluated before and after the instructional phase of this study to determine his or her spatial visualization ability and problem solving capability. Three independent measures of their spatial visualization were taken and a problem solving inventory was administered to measure their problem solving

skills. A description of each instrument is contained in this section

The Spatial Visualizations Battery

A number of studies involving spatial skills were reviewed in Chapter II. These studies had varying definitions of spatial visualization and used a variety of spatial tests. In selecting tests for this study, three criteria were considered:

- 1) The instruments should measure at least one of the two components in the definition of spatial visualization used in this study (1. the ability to recognize the relationship between parts of a given configuration and 2. the ability to mentally manipulate one or more of those parts).
- 2) The instruments should have been used in previous spatial research and should have been found to be related to spatial abilities.
- 3) The total time needed to administer the complete test battery should not exceed one class period.

Michael, Guilford, Fruchter and Zimmerman (1957) compiled various interpretations of factors in the domain of space and visualization that had been developed by previous researchers studying the nature of intelligence. They found two factors common to all the studies which they termed spatial relations and orientation and visualization. Spatial relations and orientation was defined as "an ability to comprehend the nature of the arrangement of elements

within a visual stimulus pattern" (p. 188). Visualization was defined to be the "mental manipulation of visual objects" (p. 188). These two factors correspond very closely to the two components of the definition of spatial visualization used in this study. Their study identified the Card Rotations and the Cube Comparison tests as two of the best measures of the spatial relations and orientation factor. They identified the Punched Holes test as one of the best measures of the visualization factor. Thus these three tests were selected for primary consideration as the spatial battery.

These three instruments have been used in several studies investigating spatial visualization. Carry (1968), Eastman and Carry (1975), and Webb and Carry (1975) used a derivation of the Punched Holes test in a series of studies examining how the spatial visualization aptitude interacts with two methods of teaching quadratic inequalities. The first two studies found this test to be a significant predictor of student performance on graphical treatment. Webb and Carry found that while there was some correlation between the test and student performance, it was not a significant predictor. Behr (1970) and Behr and Eastman (1975) used a derivation of the Card Rotations test and found it was also a significant predictor of student performance on graphical treatments of instruction.

Moses (1977) used all three instruments in the spatial battery in her study. She administered her battery of spatial tests to three separate populations. Each time the

Punched Hole, the Card Rotations, and the Cube Comparison tests significantly correlated with each other. Moses used factor analysis to further examine results from the spatial battery. She found that the variance was accounted for by two factors during the analysis. Eighty percent of the variance was accounted for by one of the factors, leading Moses to reject a hypothesis that spatial visualization was a decomposable aptitude. The analysis did show, however, that one spatial test, Cube Comparison, loaded more on the second factor.

The final consideration in selecting instruments for the test battery had to do with time limitations. These three tests require a total working time of 16 minutes. Thus a complete battery consisting of the three discussed spatial tests and a problem solving inventory could be administered within one class period. Therefore the Card Rotation, the Punched Hole, and The Cube Comparison tests were selected to comprise the spatial batteries.

The following is a description of the three spatial tests:

Card Rotation Test

This test was originally designed by Thurstone (1938) and was included in the Kit of Reference Tests for Cognitive Factors. The form administered to students during this study is a modified version of the original test for use with elementary school students. This modified form is from the NLSMA test battery. The stated purpose of the

Cards Rotation Test is to measure a student's ability to recognize the relationships among parts of a figure in order to identify the figure when its orientation is changed.

The test contains 14 problems. Each problem consists of a given irregular figure followed by eight other representations of that figure. The representations have been rotated from the original position, flipped over, or have been both rotated and flipped. Students mark those items which have only been rotated with a "+" and those which have been flipped over with a "-". An example is given in Figure 3.1.



Figure 3.1
Card Rotation Test Example

The students had four minutes to mark a total of 112 figures. This test has been used in several studies, such as Guay (1969) and Moses (1977). The figures in the test are two dimensional, and students can solve the problems by manipulating the figures in two dimensions.

The test was selected to measure how well the students can recognize the relationship between parts of a pattern. During the test administration, many individuals were observed physically rotating their papers. Others were

observed using hand or head motions which seemed to indicate that they were mentally manipulating the figure. Hence it would appear that this test measured both facets of the definition of spatial visualization.

Punched Hole Test

This test was created by Thurstone and was included in the Kit of Reference Tests for Cognitive Factors. The form used in this study is the modified version from the NLSMA test battery. It was modified for use with younger children than Thurstone's original form. The purpose of the Punched Hole Test is to measure the student's ability to mentally manipulate a given spatial configuration into a different configuration.

The test contains ten problems. In each problem a square piece of paper is shown after each of two or three folds. After the last fold, the paper is shown with a hole punched in it. Students must choose which of the five squares on the right contains the configuration of holes that would appear if the fold and punched square were unfolded. An example is given in figure 3.2.

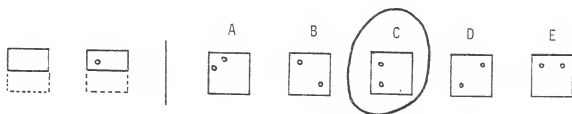


Figure 3.2
Punched Hole Test Example

Students had four minutes to mark the ten items. This test has been used in several studies, including Guay (1978), Moses (1977), and the Carry(1968), Webb and Carry (1975), and Behr and Eastman (1975) ATI series. It was selected to measure the student's ability to mentally manipulate a given configuration. The figures of the test are two dimensional, but the student must mentally manipulate the figure in three dimensions to solve the problem.

Cube Comparison Test

This test was devised by Thurstone and was included in the Kit of Reference Tests for Cognitive Factors. The form used in this study was the modified version taken from the NLSMA test battery. It was modified for use with younger children than Thurstone's original test. The purpose of the Cube Comparison Test is to measure the student's ability to recognize the relationship between parts of a given configuration.

The test contains 21 problems. Each problem consists of a pair of cubes. Each cube has three visible faces, and each face shows a letter, number, or a symbol. The directions state that no cube may have the same character on more than one face. Students must decide if the two cubes shown in the pair may represent the same cube or if they must be pictures of two different cubes. Students mark their decision under each pair, "S" if it may be the same cube, "D" if it must be two different cubes. An example is given in figure 3.3.



Figure 3.3
Cube Comparison Test Example

Students had six minutes to mark their 21 choices. This test has been used in several studies, such as Hamilton (1969) and Moses (1977). The figures in this test are representations of three dimensional objects. Students must be able to mentally visualize objects in three dimensions to solve the problems.

The test was selected to measure the students' ability to recognize relationships among the parts of a three dimensional object. During the administration of the test, several students were observed using hand motions to help themselves visualize possible manipulations of the figure. Hence it would appear that this test also measured both facets of the definition of spatial visualization.

The Problem Solving Inventory

Finding an instrument to measure the problem solving capabilities of students proved to be more difficult than selecting the spatial measures. A search of previous research failed to produce a single validated instrument appropriate for sixth grade students. There were, however, several sets of problems that had been used in other studies.

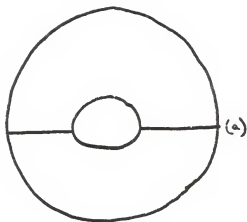
These problem sets were examined to determine vocabulary level and prerequisite mathematical knowledge. The content of the problems was also examined to determine if it met the definition of a problem contained in Chapter I and to determine if it could be classified as a spatial problem, an analytic problem, or an equally spatial and analytic problem. Many of the problem sets contained a common subset of frequently used problems.

The problems selected for the initial version were drawn from problems used by Kilpatrick (1967), Krutetskii (1971), and Moses (1977). Krutetskii and Moses had each classified the problems as spatial, analytic, or equally spatial and analytic. The classification systems used by both researchers relied on individual student interviews. Only those problems for which the two classifications agreed were considered for the inventory. Four problems from each category were chosen.

The trial problem solving inventory was field tested at Pine Grove Elementary School in Lowndes County, Georgia. A class of 26 sixth graders took the test. The distribution of the number of correct responses was approximately normal with a mean of 5.25, a median of 5, and a standard deviation of 1.87. Approximately 70% of the students scored within one standard deviation of the mean. Approximately 15% of the students scored between one and two standard deviations both above and below the mean. No student scored more than two standard deviations from the mean.

One problem on the trial inventory was identified as too easy for sixth grade students. Approximately 96% of the students solved this problem correctly. A more difficult version of the problem was selected to replace that problem. The remaining eleven problems were accepted as written in the trial inventory. The final version of the Problem Solving Inventory (see Appendix A) contained twelve problems, four in each area. The four spatial problems are listed in figure 3.4, the four analytic problems are listed in figure 3.5, and the equally spatial and analytic problems are listed in figure 3.6.

3. How many sides does a cube have? How many edges does it have?
5. A clock reads 2:50. What time will it be if the hands exchange positions?
6. Imagine a donut that is cut in half through line (a).



- Now pretend that you have picked up one half and are looking at it from the middle where it was cut. Draw what you see.
11. What kind of figure do you have if a square is rotated around one of its sides? (If you don't know its name, you can tell me something it looks like or you can draw it.)

Figure 3.4
Spatial Problems on the Inventory

4. Mr. Jones traded his horse for 2 cows. Then he traded the cows. He got 3 pigs for each cow. Then he traded his pigs. For each pig he got 6 chickens. How many chickens did he have in all?
7. Bob has a pocketful of coins that add up to \$1.05. All his coins are nickels or dimes. If there are 13 coins in all, how many nickels and how many dimes does he have?
10. A farmer has cows and chickens on his farm. He has 10 animals in all. If you count 28 feet in all, how many cows and how many chickens does the farmer have?
12. Nancy has some rabbits and some cages. When one rabbit is put in each cage, one rabbit will be left with no cage. When two rabbits are put in each cage, there will be one cage left empty. How many rabbits and how many cages are there?

Figure 3.5
Analytic Problems on the Inventory

1. There were 4 people in a race -- John, Dick, Steve, and Tom. John won the race and Steve came in last. If Tom was ahead of Dick, who came in second?
2. A fireman stood on the 4th step of a ladder, pouring water onto a burning building. As the smoke got less he climbed down 5 steps. The fire got worse so he climbed down 5 steps. Later he climbed up 7 steps and was at the top of the ladder. How many steps are in the ladder?
8. Imagine a jar that is 8 inches tall. At the bottom of the jar is a caterpillar. Each day the caterpillar crawls up 4 inches. Each night he slides back down 2 inches. How long will it take him to reach the top of the jar?
9. Susan has 4 pockets and 12 dimes. She wants to put her dimes into her pockets so that each pocket contains a different number of dimes. How many dimes does she put in each pocket to do this?

Figure 3.6
Equally Spatial and Analytic Problems
on the Inventory

Selection and Description of Population Sample

This study was conducted in the public elementary schools of Lowndes County, Georgia. No elementary school in the county had a sufficient number of students enrolled in the sixth grade to constitute an adequate sample so two comparable schools were chosen. Parker Mathis Elementary School and Lake Park Elementary School are both located outside the city limits of, but near to, Valdosta, Georgia. The majority of parents from each school work in Valdosta. The two schools are approximately equal in size. Approximately 80% of the students at each school ride buses to school. The schools have equal ethnic ratios. Their socio-economic makeup (based on the number of free and reduced price lunches served) is the same. Most of the children come from middle class families. Both principals were interested in the study and encouraged their teachers to cooperate.

Parker Mathis Elementary School had three mathematics classes, all taught by one teacher. Lake Park Elementary School had one large class taught by a two teacher team and one regular size class taught by a third mathematics teacher. These five classes constituted the sample for this study. Students from these five classes could not be randomly assigned to control and experimental groups. Intact classes

were used. This limited the degrees of freedom available during statistical analysis, but did protect against some external threats to validity. In order to minimize comparisons of instruction between students in the control group and students in the experimental group, one school was randomly selected to be the control group while the other school was designated the experimental group. Since the schools were located at opposite ends of the county and were considered rivals, there was no interaction between groups.

A total of 122 six grade students were included in the sample population. During the ten week period of the study, five students were enrolled in the classes. Nine students withdrew from the schools during the study. Only those students for whom complete pre and posttest results were obtained were included in the statistical analysis. Ten students classified as Learning Disabled or Educable Mentally Retarded were allowed to participate in the study but their test results were excluded from the statistical analysis. Therefore there were a total of 103 students whose scores were included in the statistical analysis.

The study was conducted using a sample population of sixth graders for several reasons. Students in the sixth grade have been exposed to the four operations with whole numbers. This meant more interesting problems could be selected for the Problem Solving Inventory. A second consideration for choosing sixth graders was that previous research showed sixth grade students could understand existing spatial tests. No new tests nor modified instructions would need to be developed.

The last two reasons have to do with choosing sixth graders instead of older students. McKim (1972) states that mathematical instruction in the United States is primarily analytical in nature. Students involved in the study receive eight periods of instruction in spatial visualization. This short period of instruction would have less impact on older students who are more conditioned to analytical thinking.

The final reason for choosing sixth graders relates to their prior mathematical experiences. Ability grouping in Lownders County begins in Junior high school (seventh grade).

Thus all the students in the control and experimental groups have been exposed to the same mathematical content.

Description of the Instruction Program

One of the basic purposes of education is to identify and correct any deficiency of an individual student in desirable knowledge and skills. Several researchers (e.g., McKim 1972) have indicated that the typical mathematics curriculum does not include training in spatial visualization. Further, most contend that such instruction would help correct a deficiency in the perceptual skills of American students. Thus the instruction program for this study focused on activities designed to improve the student's perceptual skills.

The instructional program lasted ten weeks. The lessons occurred during the regular mathematics instruction period. Each class was limited to 45 minutes since some students were in other rooms with different teachers for their other classes. The investigator taught all of the classes in the experimental group during each of the lessons and did all of the pre and post testing. This eliminated differences in results due to the variation in teacher effectiveness caused by differences between teachers. It introduced, however, a different threat to validity, that of the investigator influencing results. To minimize this threat, the investigator followed detailed lesson plans and was careful to follow exact testing instructions during the pre and post testing.

During the first week of the program the students were administered the three spatial abilities tests and the problem solving inventory. This was followed by eight weeks of spatial training. During the training period, each student was given several three dimensional objects to physically manipulate, was required to mentally manipulate two and three dimensional objects, and was given some practice in applying these skills to problem solving. During the tenth week the three spatial abilities tests and the problem solving inventory were again administered to the students. No prior knowledge of geometric concepts or perceptual skills was assumed in the instructional material.

Four types of activities were selected to constitute the training. The first kind of activity began with a three dimensional model being distributed to each student. Students were asked to make observations about the object. They viewed the model from many different angles. Students were then shown two dimensional representations of the object and were asked to position their model so that their visual image of the model matched the two dimensional representation. Finally students were asked to position their model in prescribed locations and draw their own two dimensional representations.

The second type of activity was a folder activity. The materials utilized for folder activities included a file folder and several 20 mm squares of construction paper. Each piece of construction paper had one or two squares,

an arrow, or an irregular shape cut out of it. A sheet was selected, shown to the students, then placed in the folder. The folder was then rotated (left or right; one, two, three, or four 90 degree turns) or flipped (top to bottom, left to right, or on either diagonal). Students were then asked to draw where the holes in the construction paper would appear when the folder was opened.

The third kind of activity in the training program was a paper folding activity. Paper folding activities required several 20 mm squares of paper. The paper was folded into halves, thirds, or fourths, and then a figure was cut out. The figures included one square, two squares, and arrow, and irregular shapes. Students viewed the paper after it had been folded and cut, and had to mentally unfold the paper and draw the positions of the holes on opened square.

The fourth type of activity in the program related visual and perceptual skills to problem solving situations. The simplest of these activities involved the student looking at pictures of stacks of blocks and deciding how many blocks were in each pile. The more advanced activities required students to translate verbal problems into pictures or diagrams.

The following is an outline of the activities conducted with students in the experimental classes in each of the eight weeks.

Week One

Distribute a cube to each student. List properties of a cube, reviewing appropriate vocabulary. Distinguish between examples and nonexamples. Formulate a definition of a cube. List examples of cubes in real life. Position cube to match two dimensional pictures of a cube.

Week Two

Distribute cubes. Review properties and the definition of a cube. Position cube to match two dimensional pictures. Students draw the cube from different angles. Folder activities were rotations with one square, two squares, and an arrow.

Week Three

Distribute cubes that are lettered with A through F. Examine letter pattern. Predict what letter is on the opposite side, top or bottom. Draw lettered cube from various angles. Folder activities were rotations with one square, two squares, an arrow, and irregular shapes, and flips with one square, two squares, and an arrow.

Week Four

Distribute a pyramid to each student. List properties, reviewing appropriate vocabulary. Distinguish between examples and nonexamples. Formulate a definition. Position the pyramid to match two dimensional pictures of the pyramid. Folder activities were rotations, flips, and combinations of rotations and flips, of one square, two squares, an arrow, and a letter.

Week Five

Distribute lettered pyramid. Review properties and the definition. Examine the letter pattern. Predict the letter on the back and bottom. Position pyramid to match two dimensional representation of the pyramid. Draw pyramids from various angles. Folder activities were rotations, flips and combinations. Paper folding activities were one fold with one square, two squares, an arrow, and irregular shapes.

Week Six

Distribute a sphere to each student. List properties, reviewing appropriate vocabulary. Distinguish between examples and non examples. Formulate a definition. List examples in real life. Draw sphere from several angles. Folder activities were rotations, flips and combinations. Paper folding activities were one and two folds, with one square, two squares, an arrow, and irregular shapes.

Week Seven

Folder activities were rotations, flips and combinations. Paper folding activities were two and three folds, with one square, two squares, and an arrow, with some cuts on the folds or the edges. Problem solving activities.

Week Eight

Folder activities were combinations of rotations and flips. Paper folding activities were two and three folds with two squares and irregular shapes, with some cuts on the folds or the edges. Problem solving activities.

The control group was pre and post tested during the same time periods as the experimental group. In the intervening eight weeks, the control group received their normal instruction following the regular mathematics curriculum. The curriculum did not include any material that dealt with spatial visualization skills.

Statistical Procedures

The purpose of this study was to examine the nature of spatial visualization and its relationship to mathematical problem solving. Questions were generated in three areas:

- (1) the nature of spatial visualization;
- (2) the relationship between spatial visualization and problem solving; and
- (3) the effect of instruction on the two areas above.

The first question considered was: Does the spatial visualization aptitude have more than one component. Three measures of the two components cited in the definition used in this study were taken. Since each test had a different number of items, each raw score was converted to a Z-score before being used. The Pearson product-moment correlation coefficient was computed between each of the three tests using pretest data from the total sample population. In addition, multiple regression analysis was done using pretest data from the total sample. Each of the three tests was selected in turn to function as the dependent variable. Scores for the remaining two tests were used as independent variables to predict the score of the dependent variable.

The second group of questions concerned the relationship between spatial visualization and problem solving. Each students' measure of spatial visualization ability, termed the spatial index, was found by summing the Z-scores of the three tests in the spatial battery. The Pearson product-moment correlation coefficient was then calculated between the students' spatial index and their problem solving performance index. Pearson product-moment correlation coefficients were also calculated between each spatial index and problem solving sub-scores on spatial problems and analytic problems.

To avoid distortion due to familiarity with the test in a test-retest situation, only pretest data were used to compute the statistics for questions in the first two areas. Using only pretest data also eliminated any confounding effects due to instruction. Since all schools in Lowndes County follow a prescribed mathematics curriculum through the sixth grade, students in the experimental and the control group had the same mathematic background prior to the study. Thus students scores from both groups could be used in answering questions from areas one and two. Questions in group three had to do with the effect of instruction. Therefore, analysis required pre and posttest data from selected subsets of all of the sample population.

Two statistics were used to examine the hypothesis concerning how instruction in visualization skills affected the subjects' spatial visualization abilities. The first

statistic was a comparison of means using the t statistic. The comparison of means was done for each of the spatial tests and the problem solving index to

- (1) compare the pretest means for the experimental group with the pretest means of the control group;
- (2) compare the pretest means of the experimental group with the posttest means of the experimental group;
- (3) compare the pretest means of the control group with the posttest means of the control group;
- (4) compare the posttest means of the experimental group with the posttest means of the control group.

The second statistic used to determine if instruction affected spatial abilities was analysis of covariance. The spatial index was used as the covariate. Group membership was control group verses experimental group.

The hypothesis concerning the effect of instruction in visualization skills on problem solving performance was investigated using analysis of covariance. The covariate was the pretest score on the problem solving inventory. Group membership was the control group verses the experimental group. The hypotheses regarding the effect on the two subsets of the problem solving inventory, spatial problems and analytic problems, were investigated using the same statistic.

The hypothesis stating that instruction does not affect high spatial students differently from low spatial students was tested by using analysis of covariance. The covariate

was the student's score on the problem solving pretest. Group membership was high spatial verses low spatial. High spatial students were defined to be those students at least one standard deviation above the mean on the spatial index. Low spatial students were those whose spatial index was one or more standard deviations below the mean. Each group consisted of approximately 15 percent of the sample.

The last hypothesis concerned whether instruction affected the problem solving performance on spatial problems differently than that on analytic problems. This was tested by a comparison of means using the *t* statistic. Since the hypothesis was concerned with the effect of instruction, only scores from the experimental group were used. A comparison of the mean of the pretest scores on the spatial problems subset with the mean of the posttest scores on the subset was done. A similar comparison was done for the subset of analytic problems.

Summary

This study was designed to investigate the nature of the aptitude called spatial visualization, its relationship to mathematical problem solving, and the effects of instruction on both of these. Three independent measures of spatial visualization were selected because they fit the definition of spatial visualization used in this study and because of results of previous studies using these tests. The three spatial tests were the Punched Holes test, the Card Rotation test, and the Cube Comparison test. A

validated problem solving inventory was not available. Problems for the inventory used in this study were drawn from problem sets used in previous research. Of the twelve problems in the inventory, four were designated spatial problems, four were analytic problems, and four were equally spatial and analytic.

The study was conducted in two elementary schools in Lowndes County, Georgia. The sample population consisted of all sixth grade students in Parker Mathis Elementary School and Lake Park Elementary School. There were 102 students in the sample population.

Students in the experimental group experienced a ten week instructional program which required students to manipulate three dimensional models, imagine the movement of the three dimensional models, practice transformations with two dimensional drawings, and do some problem solving activities. Students had one 45 minute instructional period a week in lieu of their normal mathematics curriculum. Students in the control group continued with their normal mathematics instruction.

Several statistical procedures were used in the study. Pearson product-moment correlations and multiple regression analysis were used to determine if the aptitude, spatial visualization, had one or more component parts. The Pearson product-moment correlations were also used to determine the relationship between spatial visualization and problem solving. Comparison of means t-tests and analysis

of covariance were used to determine the effects of the instructional program.

CHAPTER IV ANALYSIS OF THE DATA

Introduction

This chapter contains an analysis of the data collected during the study. The analysis provides information on three major areas. The first concerns the nature of the aptitude called spatial visualization. The second area deals with the relationship between spatial visualization and mathematical problem solving. The final area explores the effect of instruction on spatial visualization and its relationship to problem solving. Ten hypotheses were posed in the outline of the research study section in Chapter I. Each of the ten hypotheses is addressed in the analysis. Several statistics algorithms were employed. All of the programs were selected from the Statistical Analysis System package.

The Nature of Spatial Visualization

This study attempted to determine if spatial visualization is a single attribute or a composite of two or more factors. This section of the data analysis examines the results of the three spatial tests which were chosen because they reflect the definition of spatial visualization used in this study. The results in this section will be used to evaluate the first hypothesis:

1. There is not an indivisible aptitude which is called spatial visualization.

Since the treatment for the experiment group was directed towards influencing the students' spatial thinking, only the pretest scores for both the control and experimental groups were used in this part of the analysis.

The mean, standard deviation, and standard error were computed using the raw scores for the pre and post administrations of each of the three spatial tests. Table 4.1 contains these descriptive statistics for the total sample population. Table 4.2 contains the descriptive statistics for the control group only. Table 4.3 contains the descriptive statistics for the experimental group only.

Table 4.1
Descriptive Statistics for Spatial Tests
Control and Experimental Groups Combined

Test	N	Maximum Possible Score	Mean	Standard Deviation	Standard Error
Card Rotations	102	112	44.18 (55.72)	16.74 (20.20)	1.66 (2.00)
Punched Holes	102	10	4.28 (5.34)	2.36 (2.17)	.23 (.22)
Cube Comparison	102	21	9.54 (9.87)	2.36 (2.74)	.23 (.27)

(Note: Post test scores are shown in parentheses.)

Table 4.2
Descriptive Statistics for Spatial Tests
Control Group Only

Test	N	Maximum Possible Score	Mean	Standard Deviation	Standard Error
Card Rotation	52	112	41.25 (48.73)	18.01 (17.47)	2.50 (2.42)
Punched Holes	52	10	4.63 (5.04)	1.97 (2.09)	.27 (.39)
Cube Comparison	52	22	9.23 (3.67)	2.41 (2.32)	.53 (.32)

(Note: Post test scores are shown in parentheses.)

Table 4.3
Descriptive Statistics for Spatial Tests
Experimental Group Only

Test	N	Maximum Possible Score	Mean	Standard Deviation	Standard Error
Card Rotation	50	112	47.22 (62.98)	14.83 (20.44)	2.11 (2.69)
Punched Holes	50	10	3.92 (5.66)	2.69 (2.23)	.33 (.32)
Cube Comparison	50	21	9.86 (11.12)	2.30 (2.61)	.33 (.37)

(Note: Post test scores are shown in parentheses.)

The maximum possible score varied considerably among the three spatial tests, ranging from 12 to 112. To neutralize the variation in possible scores, and to ensure that each test carried equal weight in the analysis, the scores were converted to Z-scores. The mean of each conversion approached zero while the standard deviation approached one. Table 4.4 lists the actual means and standard deviations.

Table 4.4
Means and Standard Deviations of Converted Z-Scores

Variable	Mean	Standard Deviation
Cube Comparison Pretest	-.0003	.9983
Punched Holes Pretest	.0018	1.002
Card Rotations Pretest	-.0002	1.000

Two statistical tests were used to determine if spatial visualization was an indivisible aptitude or if it appeared to be composed of two or more separate factors. The first test was to calculate the Pearson product-moment correlation coefficients between each of the three tests. If the three tests were all significantly correlated to each other, then the study would have produced one indication that spatial visualization was an indivisible aptitude. If the tests were not all significantly correlated, then this would be an indication that the tests measured separate components. The correlations are listed in Table 4.5.

Table 4.5
Correlation Coefficients Between Spatial Tests

	Card Rotation	Punched Holes
Card Rotation		
Punched Holes	.0559	
Cube Comparison	.2900***	-.0152
***P < .01		

Only two of the three tests were significantly correlated. The data suggest the Cube Comparison test and the Card Rotation test measure one aspect of spatial visualization. The Punched Holes test is not significantly related to either of the other two tests. This suggests that it measures a different aspect of spatial visualization.

A second statistical procedure was applied to the spatial tests scores. Several multiple regression analyses were done with the data. Each of the three spatial tests scores served in turn as the dependent variable while the remaining two variables functioned as the independent variables. If each pair of spatial tests contributed a significant amount of the variance for the third, then this would be an indication that the three tests measured the same attribute. If one or more pairs did not contribute a significant amount, then the pair would not be a good predictor of the third. This would indicate the dependent variable measured a different component of spatial visualization than the other two variables. Table 4.6 lists the R-square, F value, and computed P values for each regression.

Table 4.6
Regression Analyses for Each Spatial Test

Dependent Variable	R-Square	F	P
Punched Holes	.0042	.21	.8111
Cube Comparison	.0351	4.23	.0122
Card Rotation	.0378	4.43	.0106

The Card Rotation test and the Cube Comparison test did not predict performance on the Punched Holes test. Thus it would again appear that the Punched Holes test measures a separate component of spatial visualization from the other two tests. Regressions with the Cube Comparison tests and the Card Rotations tests both had significant F values. Since the Punched Holes test is not correlated to nor predicted by the Card Rotation test or the Cube Comparison test, the significant F values are due to the Card Rotation test predicting the Cube Comparison test and the Cube Comparison test predicting the Card Rotation test.

Both statistical techniques produced similar results. Thus hypothesis one, there is not an indivisible aptitude which is called spatial visualization, was not rejected.

The Relationship Between Spatial Visualization and Problem Solving

The second area explored in this study was the relationship between spatial visualization and problem solving. This section of the data analysis examines the following set of hypotheses:

2. There is no correlation between spatial visualization and mathematical problem solving.
3. There is no correlation between spatial visualization and problem solving performance on spatial problems.
4. There is no correlation between spatial visualization and problem solving performance on analytic problems.

The problem solving inventory was used to measure the problem solving performance. There were twelve problems, divided into three subgroups of four: spatial problems, analytic problems, and problems both spatial and analytic. The problems were marked right or wrong on the basis of the answer. The solution process was not considered in the performance score. The means, standard deviations, and standard errors were calculated for the total sample population, the control group, and the experimental group on the whole problem solving inventory. These descriptive statistics are listed in Table 4.7.

Table 4.7
Descriptive Statistics for Problem Solving
Inventory All Classes, Control Classes,
and Experimental Classes

Population	N	Maximum Possible Score	Mean	Standard Deviation	Standard Error
Total Population	102	12	3.86 (4.47)	2.32 (2.30)	.23 (.23)
Experimental Classes	50	12	3.75 (4.67)	2.20 (2.28)	.31 (.32)
Control Classes	52	12	3.78 (4.27)	2.35 (2.33)	.33 (.32)

(Note: Post test scores are shown in parentheses.)

The means, standard deviations, and standard errors were also calculated for the spatial problem subset and the analytic problem subset. The descriptive statistics for the spatial problems subset for the total sample population, the control group, and the experimental group are presented in Table 4.8. The descriptive statistics for the total sample

population for the analytic problems subset are listed in Table 4.9. Descriptive statistics for the control group and the experimental group are also listed in Table 4.9.

Table 4.3
Descriptive Statistics for Spatial Problems Subtest
All Classes, Control Classes, and Experimental Classes

Population	N	Maximum Possible Score	Mean	Standard Deviation	Standard Error
Total Population	102	4	1.03 (1.11)	.83 (.83)	.08 (.09)
Experimental Classes	50	4	1.05 (1.24)	.83 (.79)	.19 (.11)
Control Classes	52	4	1.01 (.93)	.84 (.95)	.12 (.13)

(Note: Post test scores are shown in parentheses.)

Table 4.9
Descriptive Statistics for Analytic Problems Subtest
All Classes, Control Classes, and Experimental Classes

Population	N	Maximum Possible Score	Mean	Standard Deviation	Standard Error
Total Population	102	4	.99 (1.24)	1.01 (1.05)	.10 (.10)
Experimental Classes	50	4	.90 (1.23)	.97 (1.09)	.14 (.15)
Control Classes	52	4	1.03 (1.19)	1.04 (1.03)	.14 (.14)

(Note: Post test scores are shown in parentheses.)

To determine a single measure of each student's spatial visualization abilities, called the spatial index, the pre-test z-scores of the three spatial tests were summed. A Pearson product-moment correlation coefficient was then

computed between students' spatial index and their problem solving inventory score. Pearson product-moment correlation coefficients were also calculated between each student's spatial index and scores on the spatial problem subset and the analytic problem subset of the problem solving inventory. Again, because the treatment for the experimental group was an attempt to influence spatial abilities, only pretest data for both experimental and control groups were used. Table 4.10 lists the correlations.

Table 4.10
Correlation Coefficients Between Spatial Index
and Problem Solving Inventory and Subtests

	Spatial Index	P
Complete Inventory	.3202	.0010
Spatial Subtest	.1525	.1261
Analytic Subtest	.4331	.0001

The correlation coefficient between the spatial index and problem solving inventory score was significant at the $p < .001$ level. Thus hypothesis two, there is no correlation between spatial visualization and mathematical problem solving, was rejected. The correlation coefficient between the spatial index and performance on the spatial problem subset was only .1525, which was not significant. Thus hypothesis three, there is no correlation between spatial visualization and problem solving performance on spatial problems, was not rejected. The correlation coefficient between the spatial index and performance on the analytic

problems was significant at the $p < .0001$ level. Thus hypothesis four, there is no correlation between spatial visualization and problem solving performance on analytic problems, was rejected.

The Effects of Instruction

This section deals with the final group of hypotheses included in the study. The hypotheses are concerned with the effects of spatial instruction on a student's spatial visualization abilities and on the student's problem solving performance. The following six hypotheses were tested:

5. There is no significant difference in spatial visualization ability between the control and experimental group after the experimental group receives instruction in visualization skills.
6. There is no significant difference in problem solving performance between the control and experimental groups after the experimental group receives instruction in visualization skills.
7. There is no significant difference between control and experimental groups in performance on spatial problems after the experimental group receives instruction in visualization skills.
8. There is no significant difference between control and experimental groups in performance on analytic problems after the experimental group receives instruction in visualization skills.

9. There is no significant difference between high spatial students and low spatial students in problem solving performance after they receive instruction in visualization skills.
10. There is no significant difference between the experimental group's performance on spatial problems and their performance on analytic problems after they receive instruction in visualization skills.

Two statistical techniques were used to test the hypotheses regarding the effects of visualization instruction on visualization skills. The first statistic was comparison of group means using t-tests. The comparisons were used to answer the following questions:

1. Was there a significant difference between the control group and the experimental with respect to their performance levels on the spatial tests on problem solving prior to the instructional program?
2. Was there a significant change in the pretest to posttest performance of the control group or the experimental group on spatial tests?
3. Was there a significant difference between the control group and the experimental group with respect to their performance on spatial tests after the instructional program?

Since the two schools selected for the study were similar in socioeconomic and racial makeup, and followed the same curriculum, it was assumed that the subjects would have equal training in spatial visualization and problem solving and hence would have equal skills prior to the intervention of this study. To verify this, a comparison of means between pretest scores was done for each spatial test, the Spatial Index, the problem solving inventory, and the spatial and analytic problem subsets. The results of these comparisons are presented in Table 4.11.

Table 4.11
Comparison of Means on Pretest Scores
Between Control and Experimental Groups

Test	Group	Mean	Standard Deviation	t	n
Card Rotation	C	41.25	13.01	1.82	.0716
	E	47.22	14.88		
Punched Holes	C	4.63	1.97	-1.5269	.1303
	E	3.92	2.69		
Cube Comparison	C	9.23	2.41	1.3482	.1306
	E	9.36	2.30		
Spatial Index	C	- .1552	1.73	.3413	.4022
	E	.1641	2.05		
Problem Solving Inventory	C	3.78	2.35	- .4803	.6320
	E	3.75	2.20		
Problem Solving Spatial Subset	C	1.01	.84	.2431	.8085
	E	1.05	.83		
Problem Solving Analytic Subset	C	1.03	1.04	- .8836	.3790
	E	.90	.97		

C = Control

E = Experimental

None of the t-tests produced a significant difference. Thus it would appear that the control group and the experimental group were not significantly different in spatial or problem solving skills prior to the instructional treatment.

A comparison of means between the pretest and posttest scores was done to determine if there was a significant change in spatial skills after the instruction treatment time for either the control group or the experimental group. Table 4.12 shows the comparisons for each of the three spatial tests for both groups.

Table 4.12
Comparison of Means for Control and Experimental Groups
Between Pretest Spatial Scores and Posttest Spatial Scores

Variable	Group	Test	Mean	Standard Deviation	t	p
Card Rotation	C	Pre	41.25	13.01	2.1497	.0316
		Post	43.73	17.47		
Punched Holes	C	Pre	4.63	1.97	1.0294	.3029
		Post	5.04	2.09		
Cube Comparison	C	Pre	9.23	2.41	-1.2071	.2274
		Post	8.67	2.32		
Card Rotation	E	Pre	47.22	14.33	4.4078	.0001
		Post	62.93	20.44		
Punched Holes	E	Pre	3.92	2.69	3.5212	.0007
		Post	5.66	2.23		
Cube Comparison	E	Pre	9.36	2.30	2.5611	.0103
		Post	11.12	2.61		
C = Control						
E = Experimental						

The t-test for Card Rotation for the control group was significant at the $p < .05$ level. The Punched Holes test

showed a nonsignificant gain, while the Cube Comparison test showed a nonsignificant decrease. All three of the t-tests were significant for the experimental group, two at the $p < .001$ level. Thus it would appear that the control group's performance was not significantly changed during the time of the instructional program. The experimental group did significantly improve during the time of the instructional program.

A comparison of means on the posttest scores for each of the spatial tests was done to determine if there was a significant difference between the control group and the experimental group with respect to their performance on spatial tests after the instructional treatment. Table 4.13 lists the comparisons for the three tests.

Table 4.13
Comparison of Means on Posttest Score
Between Control and Experimental Groups

Test	Group	Mean	Standard Deviation	F	p
Card Rotation	C	48.73	17.47	3.7399	.0003
	E	62.98	20.44		
Punched Holes	C	5.04	2.09	1.4519	.1497
	E	5.66	2.23		
Cube Comparison	C	8.67	2.32	5.0072	.0001
	E	11.12	2.61		
Spatial Index	C	- .9202	1.63	5.0549	.0001
	E	.9614	2.11		
C = Control					
E = Experimental					

Two of the three spatial tests and the spatial index had significant *t* values. Only Punched Holes did not produce a significant difference, although the mean for the experimental group was higher than that of the control group. Thus it would appear that the two groups were significantly different after the instructional treatment.

The second statistical technique used to examine the effect of instruction in visualization skills on spatial ability was analysis of covariance. The students' pretest spatial index was the covariate. The posttest spatial index was the dependent variable. Group membership was the control group versus the experimental group. If group membership was significant, then changes in spatial test scores could not be attributed exclusively to differences in initial ability. Table 4.14 contains the results of the analysis.

Table 4.14
Instructional Effects on Spatial Index

Sources of Variation	SS	df	F	p
Spatial Index Pretest (Covariate)	124.95	1	54.01	.0001
Group Membership (Control vs. Experimental)	72.91	1	31.51	.0001
Covariate - Group Interaction	1.48	1	.64	.4243

The covariate-group interaction is not significant; therefore the assumptions associated with analysis of covariance are met. The covariate, the spatial index pretest

score, had a large sum of squares value and was significant at the $p < .0001$ level; thus it is an appropriate choice for a covariate. Group membership, the control group versus the experimental group, was also significant at the $p < .0001$ level. Hence it may be concluded that there is a significant difference on the spatial index posttest scores that is not due to differences in spatial ability prior to the instructional program.

Hypothesis five states there is no significant difference in spatial visualization ability between control and experimental groups after the experimental group receives instruction in visualization skills. Comparison of group means showed that the two groups were not significantly different on spatial tests prior to instruction in visualization skills. However, after such instruction, the experimental group scored significantly higher than the control group. The experimental group improved significantly on spatial tests after the instructional period while the control group did not improve after a similar lapse in time but without similar instruction. The analysis of covariance shows that the improvement was due to membership in the experimental group. Thus hypothesis five is rejected.

Hypothesis six asserts that instruction in visualization skills does not affect problem solving performance. This hypothesis was tested with two statistical procedures. The first was a series of comparisons of group means using t-tests. Table 4.11 has already presented the comparison

of the control group's and the experimental group's means on the pretest of the problem solving inventory. The non-significant difference between means, in conjunction with the other nonsignificant differences between means presented in the table, shows that the two groups were not significantly different in problem solving performance prior to the instructional treatment. Table 4.15 shows a comparison of the control group's and experimental group's means on the posttest of the problem solving inventory.

Table 4.15
Comparison of Means for Control
and Experimental Groups Between Problem
Solving Inventory Posttest Scores

Group	Mean	Standard Deviation	t	p
Control	4.27	2.33	.88	.3814
Experimental	4.67	2.28		

The results of the comparison show that there was not a significant difference between the control group and the experimental group in problem solving after the instructional program.

Table 4.16 lists the comparison between pretest and posttest means on the problem solving inventory for both the control and experimental groups.

Table 4.16
Comparison of Means for Control and
Experimental Groups Between Problem Solving
Inventory Pretest and Posttest Scores

Group	Test	Mean	Standard Deviation	t	p
Control	Pre	3.78	2.35	1.07	.2864
	Post	4.27	2.33		
Experimental	Pre	3.75	2.20	2.05	.0401
	Post	4.67	2.28		

The problem solving inventory posttest mean was not significantly different from the pretest mean for the control group. The experimental group scored significantly higher on the posttest, but only at the $p < .05$ level.

The second analysis for hypothesis six was an application of analysis of covariance. Performance on the pretest of the problem solving index was the covariate. Performance on the posttest was the dependent variable. Group membership, control versus experimental was the treatment variable. Table 4.17 gives the results of the analysis.

Table 4.17
Instructional Effects Problem Solving

Source of Variation	SS	df	F	p
Problem Solving Inventory Pretest (Covariate)	241.56	1	32.93	.0001
Group Membership (Control vs. Experimental)	.003	1	.00	.9752
Covariate-Group Interaction	2.50	1	.86	.3562

The covariate-group interaction was not significant; therefore the assumptions associated with analysis of covariance are met. The covariate, the problem solving inventory pretest score, accounted for nearly all the Sum of Squares, and was significant at the $p < .0001$ level. Thus it accounts for a significant portion of the variance and is an appropriate choice for a covariate. Group membership accounted for almost none of the Sum of Squares and was not significant. Thus it would appear that most differences in posttest scores are accounted for by differences in pretest score, and the instructional treatment did not significantly affect the posttest scores.

Hypothesis six states there is no significant difference in problem solving performance between the control and experimental groups after the experimental group receives instruction in visualization skills. Comparison of pretest and posttest means between the control and experimental groups showed that the groups were not significantly different before or after the instructional program. Although the comparison of means between pre and post test scores showed that the experimental group made a small significant improvement, the analysis of covariance did not support that the improvement was due to membership in the experimental group. Thus hypothesis six cannot be rejected.

Hypothesis seven asserts that instruction in visualization skills does not affect the problem solving performance on spatial problems. The first of two statistics used to

test this hypothesis was a series of comparison of group means using the t-test. Table 4.11 showed that there was no significant difference between the control and experimental group before the instructional treatment on any relevant measures, including differences on the spatial problem subset. Table 4.13 lists the result of a comparison of the means for the spatial problems subset posttest scores for control and experimental groups.

Table 4.18
Comparison of Means for Control and
Experimental Groups Between Spatial
Problems Subset Posttest Scores

Group	Mean	Standard Deviation	t	p
Control	.98	.95	1.49	.1392
Experimental	1.24	.79		

The results of the comparison show that there was not a significant difference between the groups in solving spatial problems following the instructional treatment.

Table 4.19 lists the comparisons between pretest and posttest means on the spatial problem subset of the problem solving inventory for both the control and experimental groups.

Table 4.19
Comparison of Means for Control and Experimental
Groups Between the Spatial Problems
Subset Pretest and Posttest Scores

Group	Test	Mean	Standard Deviation	t	n
Control	Pre	1.01	.84	-.17	.9648
	Post	.93	.95		
Experimental	Pre	1.05	.83	1.17	.2412
	Post	1.24	.79		

The comparison shows the spatial problems subset posttest mean was not significantly different from the pretest mean for either group.

The second analysis of hypothesis seven was an application of analysis of covariance. Performance on the pretest for the spatial problems subset was the covariate. Performance on the posttest was the dependent variable. Group membership, control verses experimental, was the treatment variable. Table 4.20 shows the results of the analysis.

Table 4.20
Instructional Effects on Spatial Problem
Solving Analysis of Covariance

Source of Variation	SS	df	F	n
Spatial Subset Pretest (Covariate)	129.43	1	57.01	.0001
Group Membership (Control vs. Experimental)	4.43	1	1.09	.3005
Covariate-Group Interaction	1.01	1	.76	.4233

The covariate-group interaction was not significant; therefore the assumptions for ANCOVA are met. The covariate, the pretest score on the spatial problems subset, accounted for most of the variance in the analysis. It was significant at the $p < .0001$ level and hence is an appropriate choice for a covariate. Group membership was not significant. Thus it appears that differences in pretest score accounted for most of the variation in posttest scores, and the instructional treatment did not significantly affect the posttest scores.

Hypothesis seven states there is no significant difference between control and experimental groups in performance on spatial problems after the experimental group receives instruction in visualization skills. Comparison of means between pretest and posttest scores on the spatial problems subset does not show significant differences between groups before or after the instruction in visualization skills. Neither group showed a significant change from the pretest to the posttest. The analysis of covariance supports the finding that there are no differences due to membership in the experimental group. Therefore, hypothesis seven cannot be rejected.

Hypothesis eight asserts that instruction in visualization skills does not affect the problem solving performance on analytic problems. The first of two statistical procedures used to test this hypothesis was a series of comparisons of group means using the t-test. Table 4.11

established that there was no significant difference between the control and experimental groups prior to the instructional treatment on any relevant measures, including differences on the analytic problem subset. Table 4.21 shows the results of a comparison of the means for the analytic problems subset of the problem solving inventory posttest scores for the control and experimental groups.

Table 4.21
Comparison of Means for Control and
Experimental Groups Between Analytic Problems
Subset Posttest Scores

Group	Mean	Standard Deviation	t	p
Control	1.19	1.03	.42	.6768
Experimental	1.28	1.09		

The results of the comparison show that there was not a significant difference between the two groups in solving analytic problems following the instructional program.

Table 4.22 lists the comparison between pretest and posttest means on the analytic problems subset for both the control group and the experimental group.

Table 4.22
Comparison of Means for Control and
Experimental Groups Between Analytic Problems
Subset Pretest and Posttest Scores

Group	Test	Mean	Standard Deviation	t	p
Control	Pre	1.08	1.04	.54	.5880
	Post	1.19	1.03		
Experimental	Pre	.90	.97	1.84	.0625
	Post	1.28	1.09		

The comparison shows that analytic problems subset posttest means were not significantly different from the pretest means for either group.

The second analysis of hypothesis eight was an application of analysis of covariance. Performance on the pretest for the analytic problems subset was the covariate. Performance on the posttest was the dependent variable. Group membership, control versus experimental, was the treatment variable. Table 4.23 shows the results of the analysis.

Table 4.23
Instructional Effects on Analytic Problem
Solving Analysis of Covariance

Source of Variation	SS	df	F	p
Analytic Subset Pretest (Covariate)	121.25	1	53.13	.0001
Group Membership (Control vs. Experimental)	7.35	1	2.05	.1782
Covariate-Group Interaction	1.40	1	.56	.4315

The covariate-group interaction was not significant; therefore the assumptions for ANCOVA are met. The covariate, the pretest score on the analytic problem subset, was significant at the $p < .0001$ level and thus is an appropriate choice for a covariate. Group membership was not significant. Therefore it appears that differences in pretest scores account for most of the variation in posttest scores, and the instruction program did not significantly affect the posttest scores.

Hypothesis eight states there is no significant difference between control and experimental groups in performance on analytic problems after the experimental group receives instruction in visualization skills. Comparison of the means for pretest and posttest scores on the analytic problems subset does not show significant differences between groups before or after the instruction in visualization skills. Neither group demonstrates a significant change from the pretest to the posttest on analytic problems. The analysis of covariance supports the findings of no significant differences due to membership in the experimental group. Therefore hypothesis eight cannot be rejected.

Hypothesis nine asserts that instruction in visualization skills does not affect the problem solving performance of high spatial students differently from that of low spatial students. For the purpose of this study, high spatial students were defined to be those students whose spatial

index was at least one standard deviation above the mean. Low spatial students were those whose spatial index was one standard deviation below the mean. Since the hypothesis concerns how instruction affects student performance differently only students in the experimental group were considered. A total of 18 students, seven classified as low and eleven classified as high, were included in the analysis. Analysis of covariance was used on the data, with pretest scores on the problem solving inventory as the covariate, the posttest score on the inventory the dependent variable, and group membership, high spatial versus low spatial, the treatment variable. Table 4.24 shows the results of the analysis.

Table 4.24

Instruction Effects on High and Low Spatial Students

Sources of Variation	SS	df	F	p
Problem Solving Inventory Pretest (Covariate)	16.33	1	12.09	.0037
Group Membership (High Spatial vs. Low Spatial)	2.45	1	1.82	.1987
Covariate-Group Interaction	.34	1	.25	.6257

The covariate-group interaction was not significant so assumptions for ANCOVA are satisfied. The covariate, pretest scores on the problem solving inventory was significant at the $p < .001$ level and is an appropriate covariate choice. Group membership was not significant. Thus it appears that group membership in the high spatial group or the low

spatial group does not explain a significant amount of variance in the posttest problem solving inventory score.

Therefore hypothesis nine, there is no significant difference between high spatial students and low spatial students in problem solving performance after they receive instruction in visualization skills, cannot be rejected.

Hypothesis ten asserts that instruction in visualization skills does not affect problem solving performance on spatial problems differently than that on analytic problems. Since the hypothesis concerns how instruction affects performance differently on certain kinds of problems, only data from the experimental group were considered. The hypothesis was tested by comparison of means using the t-test.

The test compared the mean of the spatial subset gain (posttest score minus the pretest score) and the mean of the analytic subset gain (posttest score minus pretest score).

Table 4.25 lists the results.

Table 4.25
Instruction Effects on Problem Solving
Performance Gain for Spatial Problems
and Analytic Problems

	Mean	Standard Deviation	t-value	p
Spatial Problems Subset	.19	.82	2.56	.0134
Analytic Problems Subset	.38	1.05		

The t-value is significant at the $p < .02$ level. This indicates that the mean gain on analytic problems is significantly greater than the mean gain on spatial problems.

Therefore hypothesis ten, there is no significant difference between the experimental group's performance on spatial problems and their performance on analytic problems after they receive instruction in visual skills.

Summary

Some of the hypothesized statements regarding the nature of spatial visualization, the relationship between spatial visualization and mathematical problem solving, and the effects of instruction on spatial visualization and that relationship were rejected:

2. There is no correlation between spatial visualization and mathematical problem solving performance.
4. There is no correlation between spatial visualization and problem solving performance on analytic problems.
5. There is no significant difference in spatial visualization ability between the control and experimental groups after the experimental group receives instruction in visualization skills.
10. There is no significant difference between the experimental group's performance on spatial problems and their performance on analytic problems after they receive instruction in visualization skills.

Spatial visualization appears to be a composite of more than one factor, it is correlated to problem solving skills, and it is an aptitude which can be modified through training.

A discussion of the results of this analysis and their implications is presented in Chapter V.

CHAPTER V SUMMARY CONCLUSIONS AND IMPLICATIONS

This chapter is divided into seven sections. The first section reviews the objectives and methodology of the study. The second section provides a summary of the results presented in Chapter IV. This review provides the basis for the third section, a discussion of the conclusions made from the results. The fourth and fifth sections deal with the implications from this study for future research and for curriculum modifications. The sixth section discusses the limitations of the study. The final section is a chapter summary.

Review of the Study

This study was designed to investigate the nature of spatial visualization, its relationship to mathematical problem solving, and the effect of instruction on both the aptitude and the relationship. Three aptitude tests, Card Rotation, Cube Comparison, and Punched Holes, were selected to measure the student's spatial abilities. These tests were selected because they matched the study's definition of spatial visualization and because of the results of previous studies that incorporated them (Moses, 1977; Carry, 1962).

A problem solving inventory was used in addition to the spatial tests to determine the relationship between problem solving and spatial visualization. An existing, validated

inventory was not available so problems were drawn from other promising studies (Kilpatrick, 1967; Krutetskii, 1971; Moses, 1977). The inventory was divided into three subsets of four problems each: 1) spatial problems, 2) analytic problems, and 3) problems considered equally spatial and analytic.

The study was conducted at Parker Mathis Elementary School and Lake Park Elementary School in Lowndes County, Georgia. The sample population was composed of all the sixth grade students at the two schools. Students at Parker Mathis Elementary School were the experimental group while students at Lake Park Elementary School were the control group. Complete data were collected on 102 students.

The instructional program lasted ten weeks. The first and last weeks were devoted to data collection. In the intervening eight weeks, the experimental group had one 45 minute period per week devoted to developing visualization skills in lieu of their regular mathematics instruction. Students manipulated three dimensional models, imagined the movement of three dimensional models, practiced transformations with two dimensional drawings, and experienced some problem solving activities. The control group continued their regular mathematics curriculum.

Several statistics were used to evaluate the data. The first area considered was the nature of spatial visualization. Pearson product-moment correlation coefficients and regression analysis were used to determine if spatial

visualization was a single aptitude or if it was composed of more than one factor. Pearson product-moment correlation coefficients were also used to determine if spatial visualization was related to problem solving. Finally, comparison of group means t-tests and analysis of covariance were used to examine the effect of instruction in visualization skills on spatial ability and various aspects of problem solving.

Summary of the Results

This study attempted to answer ten questions covering three areas. The first topic examined was the nature of the aptitude called spatial visualization. The second area probed during the study was the relationship between spatial visualization and mathematical problem solving. The third area investigated was the effect of an instructional program on spatial visualization and on the relationship between spatial visualization and problem solving. This section summarizes the results of the study in an effort to answer the questions posed in each area.

The Nature of Spatial Visualization

To explore the nature of spatial visualization, the study attempted to answer the following question:

1. Does the aptitude called spatial visualization consist of a single ability or does it have two or more component parts?

Two statistics were used to determine if spatial visualization was a single aptitude or composed of more than one

component. Pearson product-moment correlation coefficients were computed between each of the three spatial tests. Card Rotation and Cube Comparison were significantly related at the $p < .01$ level. The Punched Holes test was not significantly correlated to either of the other two tests.

A series of multiple regression analyses produced supporting results. Each of the three tests functioned in turn as the dependent variable while the other two tests were tested to see how well they predicted the score on the dependent test. When Punched Holes was the dependent variable, the amount of variance accounted for by Card Rotation and Cube Comparison was not significant. Both Card Rotation and Cube Comparison had significant F-values when they were the dependent variable. Since Punched Holes is uncorrelated with Card Rotation and Cube Comparison, and not predicted by them, the variance accounted for in Card Rotation must come from Cube Comparison and the variance in the Cube Comparison from Card Rotation. Thus it appears that Punched Holes measures a separate factor. Therefore the answer to the first question is that spatial visualization is not a single ability.

The Relationship Between Spatial Visualization and Problem Solving

A series of three questions was posed to examine the relationship between spatial visualization and mathematical problem solving. They were

2. Is there a relationship between a student's spatial index and his mathematical problem solving performance.

3. Is there a relationship between a student's spatial index and his problem solving performance on spatial problems?
4. Is there a relationship between a student's spatial index and his problem solving performance on analytic problems?

To answer these questions, a single measure of spatial visualization ability, called a spatial index, was created for each student by converting the scores on each of the three spatial tests to z-scores and finding their sum. This ensured each test was equally weighed in the analysis. The following Pearson product-moment correlations were derived using the spatial index and the scores from the problem solving test:

- A. Spatial visualization ability correlates significantly with mathematical problem solving performance at the $p < .001$ level.
- B. Spatial visualization ability and performance on spatial problems are not significantly correlated.
- C. Spatial visualization ability correlates significantly with performance on analytic problems at the $p < .001$ level.

Therefore it appears that the answer to questions two and four is yes, there are relationships between the students' spatial index and both their total problem solving performance and their analytic problem solving performance. The answer to question three, however, is no; there is not

a relationship between the spatial index and spatial problem solving performance. The correlation in A was expected and has support in the literature. The correlations in B and C were not expected and are discussed further in the conclusions section.

The Effects of Instruction

Six questions were posed to determine the effect of instruction in visualization skills on the measure of spatial visualization aptitude and on its relationship to problem solving. Two statistics, comparison of means t-test and analysis of covariance, were used in an attempt to answer these questions.

The first question in this series was

5. What effect will instruction in visualization skills have on a student's spatial visualization aptitude?

The results of analysis showed that the experimental group demonstrated significant gain on all three spatial tests after the instruction program. The control group had a significant gain on Card Rotation test, a slight but non-significant gain on Punched Holes test, and a small, nonsignificant decrease on the Cube Comparison test. Although there was no significant difference between groups prior to instruction, comparison of posttest scores between groups showed the experimental group scored significantly higher on the Card Rotation test, Cube Comparison test, and the spatial index. On the third test, Punched Holes, the

experimental group scored higher, but it was not a significant gain. Analysis of covariance supported the conclusion that the experimental group performed significantly better on the posttest, even when initial differences are removed. Thus it would appear that instruction in visualization skills improved the students' spatial visualization skills.

The next three questions dealt with the effect of instruction on problem solving. They were

6. What effect will instruction in visualization skills have on problem solving performance?
7. What effect will instruction in visualization skills have on the problem solving performance on spatial problems?
8. What effect will instruction in visualization skills have on the problem solving performance on analytic problems?

Comparisons of group means from pretest to posttests for both groups showed only one t-test indicating a significant change. The experimental group demonstrated a significant gain, at the $p < .05$ level, on the total problem solving inventory. The control group did not show any significant gains, and in fact registered a small, nonsignificant decrease on the spatial problems subset.

Analyses of covariance were done to determine if group membership made a significant difference on posttest scores when the whole problem solving inventory, the spatial problems subset, and the analytic problem subset were used as

covariates. In each case, when the effects due to scores on the pretest of a problem set were removed, group membership was not significant. That is, once the students' initial ability levels were equalized, students did not show a significant gain in problem solving skills from instruction in visualization skills. With only the small significant gain on the total problem solving inventory on the comparison of group means and no support from analysis of covariance, it was not possible to conclude that the instruction in visualization skills had a significant impact on problem solving. The section on conclusion discusses some possible reasons for the lack of gain.

The next question concerning the effects of instruction was

9. Will instruction in visualization skills affect the problem solving performance of high spatial students differently than that of low spatial students?

An attempt to answer this question was made by using analysis of covariance. Only scores from students in the experimental group were used since the control group did not receive the instruction. After the variation due to scores on the pretest of the problem solving inventory was removed, group membership in high or low spatial groups was not significant. Thus performance on the problem solving inventory pretest was a better predictor of the posttest problem solving inventory score than high or low spatial

performance. Therefore it is not possible to say that instruction in visualization skills affects high spatial students differently than low spatial students.

The final question on the study was

10. Will instruction in visualization skills affect problem solving performance on spatial problems differently than that on analytic problems?

To answer this question, pretest and posttest scores on the analytic and spatial problem subsets were compared for students in the experimental group to compute the amount of gain for each student. The gain on the analytic problems subset was significantly higher than on the spatial subset. This result was surprising, but consistent with the correlations between spatial scores and problem solving reported earlier.

Conclusions and Discussion

The majority of conclusions discussed in this section were arrived at primarily through the analysis presented in Chapter IV. In addition, some conclusions reached during the course of the study, based on student and teacher reactions to the tests and the instructional program, are included. The discussion will indicate statistical support for such conclusions when possible.

Conclusion About the Nature of Spatial Visualization

1. Spatial Visualization is not a single ability.

The lack of correlations between spatial tests reported in Chapter IV indicated that the three spatial tests

utilized in this study were not measuring the same attribute. The tests were chosen because they reflected the definition of spatial visualization: the ability to recognize the relationship between parts of a given visual configuration and to manipulate one or more of those parts. Several studies are presented in Chapter II that agree with this conclusion (e.g., Michael et al., 1957; Guilford et al., 1952), but there are also several recent studies that found an indecomposable spatial aptitude (e.g., Moses 1977; French, 1965).

2. Spatial visualization is not a skill taught in current mathematics curriculum.

Little evidence of spatial development or activities was found during examinations of elementary mathematics tests prior to the study. Even concepts in geometry, which could have been approached spatially, were handed in an analytic mode. Student reactions during the presentation of material and the desire of the regular classroom teacher to participate in the training activities contributed to this conclusion. Some students stated they had not handled cubes, pyramids, or spheres during a mathematics class before the training began.

Results on the three spatial tests would further support this conclusion. Means on the pretests ranged from 37% correct to 47% correct. After eight 45 minute lessons, posttest means for the experimental class rose to 53% to 56% correct.

This conclusion is also supported by some of the researchers cited in Chapter II. McKim (1972) believes spatial opportunities are, at best, only available in the primary grades. Thurstone (1950) also has stated that most instruction in elementary grades is aimed at the analytic thought process. Thus the classroom observations that initiated this conclusion appear to be supported by research results of this study and others.

Conclusions About the Relationship Between Spatial Visualization and Problem Solving

3. Spatial visualization is a good predictor of general problem solving.

Statistical results in Chapter IV show a strong correlation between the total problem solving inventory and the spatial index. The highest correlation was with the analytic problems subset, while the lowest correlation, and only nonsignificant one, was the spatial problems. The number of problems on the whole inventory was limited by the amount of time students would have to complete the pre and posttest battery. Thus the number allotted to each of the three kinds of problems, spatial, analytic and equally spatial and analytic, was very small. The means for the groups ranged from 22% to 31% correct. This sample of student work is not large enough to give an accurate picture of abilities.

Three other points may have contributed to these results. Most classroom instruction is in an analytic framework. Therefore, successful students would be those with

good analytic skills. Students who are successful are more likely to be receptive to unfamiliar challenges. These students would be more receptive to the unfamiliar spatial tests and would perhaps do better. A second possibility is that high spatial ability compensates for some lack in analytic skills. Thus problems that would ordinarily be expected to be solved analytically were actually solved using spatial skills. In the same vein, students may have been trying to solve spatial problems using analytic techniques. The problems were specifically chosen that did not lend themselves to that approach.

Conclusions About Effects of Instruction

4. Spatial visualization is a trainable attribute.

Several research studies were presented which debated whether males or females had better spatial skills. Some researchers suggested it was an attribute whose potential was determined genetically. McKim responded with "Whatever the inheritance, the unrealized potential for visual development is great" (1972, p. 25). The data in this study clearly support a significant gain in spatial performance after training. Eight 45 minute classes during an eight week session do not provide much time for training yet most students made significant gains in spatial scores.

5. Visualization instruction does not significantly affect problem solving performance.

A review of research shows that mixed results have been obtained in the past. Researchers who have used geometry

as their content base have been more successful than those who used algebra. One general problem has been the absence of a validated problem solving inventory. Researchers have chosen a wide variety of instruments on which to base a measure of problem solving performance.

This study used 12 nonroutine mathematical problems which required only simple computation skills once a solution method had been determined. Students reactions to the inventory and to problems posed during the instructional period indicated a lack of experience with nonroutine problems. Students were not persistent in efforts to solve the problems. They quickly gave up and stopped work or asked for hints to a solution.

The short training period for spatial thinking was not sufficient time to overcome several years of training to approach problems analytically. Again the small number of problems completed during the testing period may not have been an adequate sample.

There is also the question of transfer. The majority of the instructional time was spent on visualization training. Problem solving activities were confined to a portion of the final three weeks. This may not have been sufficient time or contained enough models that would allow students to apply their enhanced spatial skills to problem solving solutions.

6. Instruction affects performance on analytic problems differently than spatial problems.

Examining the students scores, more gains were noted on the analytic problems. This would support an earlier statement that spatial skills may somehow compensate for skills required to solve analytic problems. Better analytic scores for the experimental group is also consistent with earlier correlation results. Student performance on analytic problem solving was unexpectedly correlated with spatial skills while performance on spatial problems was not. Spatial skills were improved through the training program. Analytic problem solving improved more than spatial problem solving. Examining mean gains, students gained significantly more on analytic problems than on spatial problems. Recall, however, that the actual gains means were so small (.19 to .33), and the number of problems with which to work were so limited, that data may not have reflected true changes in skill levels.

Limitations

Internal and external threats to validity were considered during the design of the study. Some threats could not be avoided and so an effort was made to minimize those threats. This section will expand upon the limitations addressed in Chapter I and identify threats which could not be avoided.

The population of this study came from a public school system. This forced certain parameters on the study. First, students had to be instructed in intact groups and could not be randomly assigned to experimental and control

groups. Students were not ability grouped in this school system and were not assigned to classes on the basis of any formal system. The two schools selected for use in the study drew from similar populations. Thus the intact group would not appear to be a threat to validity.

A second limitation imposed by operating in a school was the amount of time available for instruction. Student classes were 45 minutes long. At the end of that time students changed classes, and another group entered the room. Thus instruction had to stop, regardless of where the interruption occurred. During the study, this caused several promising discussions to be terminated prematurely.

The instructional program included eight instructional periods. This was not sufficient time to overcome the influence of the analytical instruction the students had received. The amount of improvement demonstrated after that short time would indicate promising results if the instruction could be sustained over a longer period. Ten weeks elapsed between administrations of the test batteries. This appeared to be sufficient time to overcome any possible test-retest interaction. The ten week period did contribute to the mortality rate in the study. Although the number of participants at the experimental school remained relatively constant at 62, only 50 students had complete pretest-posttest battery scores.

The number in the sample population was limited by the number of students enrolled in the two schools. A larger

sample would have required that more schools would be involved and that some of the instruction would have been done by someone other than the researcher. This would have introduced variability due to different teaching styles. Having any person, other than the regular classroom teacher, do the instruction may have influenced the results due to the Hawthorne effect. However, the researcher was not completely unknown to the sample population. The researcher had done sample lessons in many of the lower grade classes the previous year. The students were not aware that this teaching was any different from those appearances.

The testing instruments available to measure spatial visualization and problem solving were also limiting factors. It appeared that the spatial tests were not measuring the same ability for all students. In Cubes Comparison, for example, there seemed to be three methods of selecting an answer. Some students mentally rotated the whole cube to see if a position to match the second picture could be found. Other students tried to relate the sides of the cube to their own body position, then move the paper to match that position for the second cube. A third group appeared to approach the problem in an analytic fashion, dissecting the cube and comparing component parts. Similar strategies were applied to the other two spatial tests as well. Thus, spatial scores reflected different abilities. A pure spatial test, highly resistant to analytic solution, is needed.

The problem solving inventory presented similar problems. Although only problems classified as spatial or analytic by more than one researcher were utilized, many students solved spatial problems analytically, while other solved analytic problems by employing some spatial techniques. Thus scores on the spatial problems subset did not represent spatial abilities for some students, while analytic problems subset scores did not represent analytic abilities for other students.

The results of this study should be accepted cautiously. The results were obtained for one sample of sixth graders. Other test scores would indicate this was an average group. Thus, there is no reason to suspect the results could not be generalized to the total sixth grade population. However, differences may occur in other age groups. In the same vein, the problem selected contained mathematical content. The results may not generalize to other content areas even for students in the sixth grade.

Implications for Future Research

Many mathematicians and mathematic educators continue to believe that spatial ability plays a large role in understanding mathematics. This research indicated a significant relationship between spatial visualization and problem solving, but was not able to show that improving spatial visualization scores improved problem solving scores. This could, in part, be attributed to weaknesses in the spatial tests. Fruchter (1954), in a survey of earlier spatial research

studies, pointed out that the same spatial tests have not loaded consistently on the same factor during factor analysis. This inconsistency is further supported by contradictory correlation coefficients in different studies. A study by Michael, Zimmerman and Guilford (1957) and this study found a lack of correlation between the Punched Holes test and either the Card Rotation test or the Cube Comparison test. Moses (1979) and French (1965) both found a significant correlation between tests. Thus the most critical area for future research is developing spatial tests which satisfy the definition of spatial visualization, are consistently correlated and load uniformly on the same factor, and are resistant to being solved analytically.

It may be necessary to do further research to clearly define what constitutes spatial thinking prior to work on spatial tests. The review of research on spatial visualization in Chapter II presents many differing definitions of spatial visualization. A standard definition of spatial visualization must be used by researchers. If spatial visualization is to be a separate aptitude, items that would also appeal to analytical abilities must be excluded. Research into the functioning of left and right brain hemispheres holds promise in this area. If researchers are able to establish that spatial thinking and analytic thinking are on opposite sides of the brain, research could be done to establish what types of situations stimulate analytic and spatial thinking.

Another area of research that would help clarify the conflicting results of past spatial-problem solving studies is identifying the involvement of spatial visualization in different types of mathematical problems. It is likely different topics within mathematics require different amounts of spatial skills and perhaps even different skills. It is also possible different age groups require different spatial abilities to solve the same problem. For example, a kindergarten child may need visualization skills to picture the joining of sets that is the typical presentation of addition problems. A fifth grade student would be expected to use rote memory for such an exercise.

The recent advent of the microcomputer in the classroom offers a new opportunity to study spatial skills. Programs developed by the Minnesota Educational Computing Consortium in art and aesthometry demonstrate the computer's ability to show perspective and draw three dimensional objects. Other programs have been developed which can demonstrate the rotation of a three dimensional object along any of the three main axes. Programs of this type, used in conjunction with real three dimensional objects, would aid a student's understanding of movement component of spatial visualization. Unique opportunities exist with the computer to develop spatial tests which utilize the fluid motion possible on a computer screen and lessen the possibility of employing analytical skills to solve the problems.

Once suitable testing instruments have been identified, research similar to this study, but carried out over a longer time period, would be instructive. Other modifications could include a greater emphasis on transfer during the whole training program and more frequent instructional periods. One difficulty of working with problem solving is the general negative attitude of the average student to any problem that is written in a verbal form. Longer instruction, in addition to perhaps providing greater skill development, might help overcome that attitude. It would also be interesting to discover if there was a floor at which spatial abilities began to affect problem solving. Although this study was not able to determine that high spatial students were affected differently than low spatial students, high students in this study were defined as those at least one standard deviation above the group average, and the average in this sample was low. Thus, students who do have good spatial skills might react differently to instruction.

Implications for Curriculum Changes

The major implication for curriculum change is the desirability of teaching spatial skills. The curriculum already includes so many topics that it is difficult for most teachers to cover what is already expected. It is not reasonable to continue to simply add more topics. However, the results of this study and others showing strong correlations between spatial skills and problem solving argue for their inclusion. One can avoid the arguments concerning which

skills may no longer be essential and still include spatial skills by modifying some current topics. Geometry in the elementary schools is an excellent choice for the inclusion of spatial skills. An informal approach to geometry utilizing more concrete models would be more suited to the child's developmental stage. Elementary teachers at all grade levels, not just primary, should be encouraged to use manipulatives when introducing concepts.

Students enjoyed drawing the models in different positions and seemed to gain more when instruction involved more than just mental skills. Mathematical topics should be explored to see which ones can be modified to allow students to use the maximum number of senses. Crossing subject discipline lines to have mathematical concepts demonstrated through science and geometric models emphasized in art could not only increase spatial skills but increase student motivation as well.

Summary

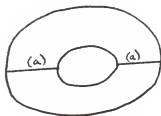
This study explored the nature of spatial visualization, its relationship to problem solving, and the effect of instruction of both. The findings suggest that the spatial visualization aptitude consists of two or more component parts. They further suggest that spatial visualization skills can be improved through training. The students spatial visualization skill level appears to be a good predictor of problem solving performance. However, improving spatial visualization skills through training did not significantly improve problem solving performance.

APPENDIX A
PROBLEM SOLVING INVENTORY

1. There were 4 people in a race --- John, Dick, Steve and Tom. John won the race and Steve came in last. If Tom was ahead of Dick, who came in second?
2. A fireman stood on the 4th step of a ladder, pouring water onto a burning building. As the smoke got less he climbed up 4 more steps. The fire got worse so he climbed down 5 steps. Later he climbed up 7 steps and was at the top of the ladder. How many steps are in the ladder?
3. How many sides does a cube have? How many edges does it have?
4. Mr. Jones traded his horse for 2 cows. Then he traded the cows. He got 3 pigs for each cow. Then he traded his pigs. For each pig he got 6 chickens. How many chickens did he have in all?

5. A clock reads 2:50. What time will it be if the hands exchange positions?

6. Imagine a donut that is cut in half through line (a).



Now pretend that you have picked up one half and are looking at it from the middle where it was cut. Draw what you would see.

7. Bob has a pocketful of coins that add up to \$1.05. All his coins are nickels or dimes. If there are 13 coins in all, how many nickels and how many dimes does he have?
8. Imagine a jar that is 3 inches tall. At the bottom of the jar is a caterpillar. Each day the caterpillar crawls up 4 inches. Each night he slides back down 2 inches. How long will it take him to reach the top of the jar?

9. Susan has 4 pockets and 12 dimes. She wants to put her dimes into her pockets so that each pocket contains a different number of dimes. How many dimes does she put in each pocket to do this?

10. A farmer has cows and chickens on his farm. He has 10 animals in all. If you count 28 feet in all, how many cows and how many chickens does the farmer have?

11. What kind of figure do you have if a square is rotated around one of its sides? (If you don't know its name, can you tell me something it looks like or can you draw it?)

12. Nancy has some rabbits and some cages. When one rabbit is put in each cage, one rabbit will be left with no cage. When two rabbits are put in each cage, there will be one cage left empty. How many rabbits and how many cages are there?

APPENDIX B
SPATIAL VISUALIZATION TRAINING ACTIVITIES
WITH THREE DIMENSIONAL MODELS

Students worked with cubes, pyramids, and spheres during the instructional program. Each student was given the appropriate three dimensional model during the lessons and asked to describe its characteristics, move it so that their view of it matches a given picture of the model, and, finally, imagine and draw the view of it when the model was in a prescribed position. Some discussion of perspective and drawing conventions were included in the discussions.

Cubes

Instruction began with a discussion of what is a cube. Students were then instructed to move the cube to various positions relative to their body and describe what they saw. Students then were asked to position their cube to match two dimensional pictures of a cube. Figure B.1 contains samples of the drawings students tried to match.



Figure B.1
Sample of Two Dimensional Pictures

After students had practice moving their cube to match a given picture, they were told to imagine what they would see if the cube were moved to a particular location. There was a discussion of perspective and drawing conventions during this exercise. Students were told to sketch what they thought they would see. They checked their sketch by moving their cubes to the position and comparing the view to the sketch.

The next stage of instruction involved using a lettered cube. Figure B.2 shows the lettering pattern.

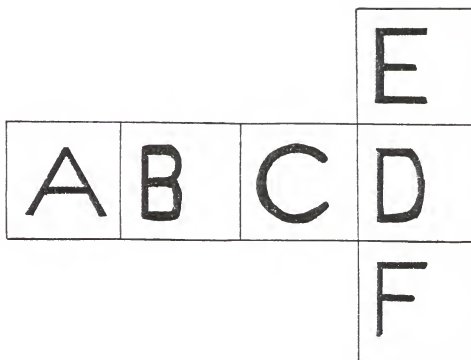


Figure B.2
Lettering Pattern for Cube

Students again went through the exercises of positioning their cube to match a two dimensional drawing, this time with letters on the drawing. Figure B.3 shows two examples of such drawings.

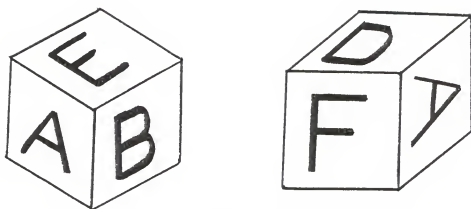


Figure B.3
Two Dimensional Drawings Which Match
Lettering Pattern

The concluding work with lettered cubes involved students looking at two dimensional drawings of a lettered cube to determine if they could position their cube so that the perspective and the letters match the drawing. Figure B.3 above shows a sample of the possible drawings while figure B.4 shows a sample of those that were not possible.

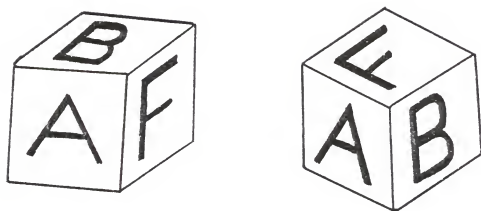


Figure B.4
Two Dimensional Drawings Impossible
with Lettering Pattern

Pyramids

Instruction began with a discussion of what is a pyramid. Students were then instructed to move the pyramid to various positions relative to their body and describe what they saw. Students then were asked to position their pyramid to match two dimensional pictures of a pyramid. Figure B.5 contains samples of the drawings students tried to match.

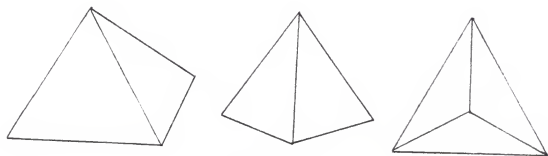


Figure B.5
Sample Two Dimensional Pictures

After students had practice moving their pyramid to match a given picture, they were told to imagine what they would see if the pyramid were moved to a particular location. There was a discussion of perspective and drawing conventions during this exercise. Students were told to sketch what they thought they would see. They checked their sketch by moving their pyramids to the position and comparing the view to the sketch.

The next stage of instruction involved using a lettered pyramid. Figure B.6 shows the lettering pattern.

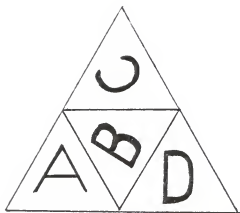


Figure B.6
Lettering Pattern for Pyramid

Students again went through the exercises of positioning their pyramid to match a two dimensional drawing, this time with letters on the drawing. Figure 3.7 shows two examples of such drawings.

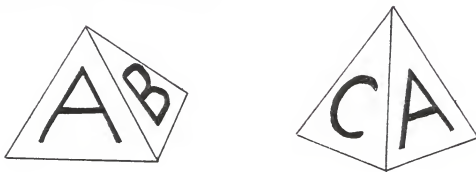


Figure B.7
Two Dimensional Drawings Which
Match Lettering Patterns

The concluding work with lettered pyramids involved students looking at two dimensional drawings of a lettered pyramid to determine if they could position their pyramid so that the perspective and the letters match the drawing. Figure B.7 above shows a sample of the possible drawings while figure B.8 shows a sample of those that were not possible.



Figure B.8
Two Dimensional Drawings Which
Can Not Match Lettering Pattern

Spheres

Instruction concerning the sphere began with a discussion of the properties of a sphere. Students were then instructed to move their sphere to various positions relative to their body. After several moves, students were able to deduce that the sphere would appear the same from all angles. Some professional drawings of spheres showing the shading techniques used to give two dimensional drawings the appearance of three dimensions were displayed and the drawing conventions were discussed.

APPENDIX C
SPATIAL VISUALIZATION TRAINING
WITH FILE FOLDERS

The materials necessary for these exercises included a file folder that had been trimmed to be a square and several sheets of paper that had been trimmed to slightly smaller squares. Students were given a piece of paper showing twenty small squares in which they sketched their responses.

Each of the square sheets of paper had a design cut out of it. Figure C.1 contains several examples of designs used during the training.

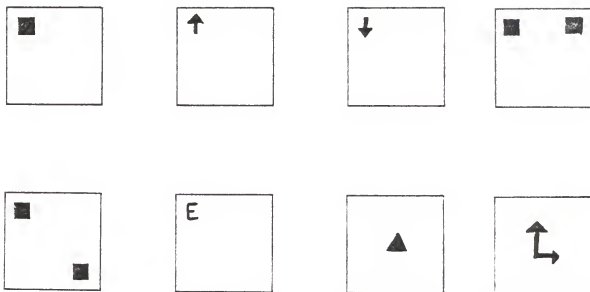


Figure C.1
File Folder Designs

One of the designs was put in the folder with the students observing its initial position. The folder was then closed and manipulated. Students drew how they thought the design would be positioned when the folder was opened.

The file folder was manipulated in one of two ways. In initial exercises the folder was rotated. To avoid any confusion based on vocabulary, the rotation was referred to as a turn. One ninety degree rotation was called one turn, two turns was a rotation of 180 degrees, three turns was a rotation of 270 degrees, and four turns was a rotation of 360 degrees. Figure C.2 shows how the designs were affected by turns.

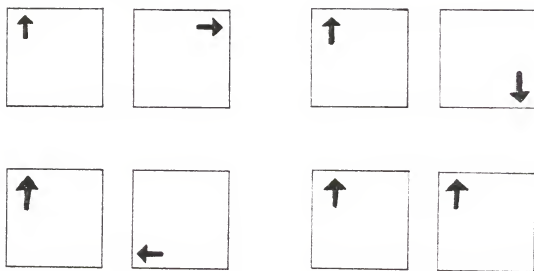


Figure C.2
File Folder Turns

After students had practiced with turns, a second method of manipulation was introduced. The folder was flipped along its axes of symmetry. Figure C.3 shows how the designs were affected by flips.

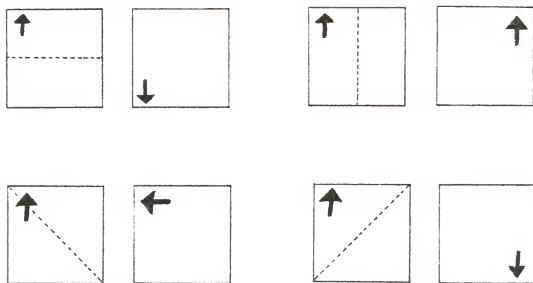


Figure C.3
File Folder Flips

Once students were familiar with both methods of manipulation, the two were mixed together in multiple combinations.

APPENDIX D
SPATIAL VISUALIZATION TRAINING
WITH PAPER FOLDING

The material required for these activities consisted of square sheets of paper. The sheets were folded before the classes and designs were cut through all layers. Students were shown at the beginning of each lesson how the paper was folded. They were then shown the folded sheets with the designs cut out and asked to imagine the paper being unfolded and sketch where the designs were.

The exercises began with papers that had either one vertical or one horizontal fold. Figure D.1 shows an example of the folded paper and the unfolded paper.



Figure D.1
Paper Folding Patterns

The exercises then progressed to papers with two folds. Figure D.2 shows several examples of two folds.

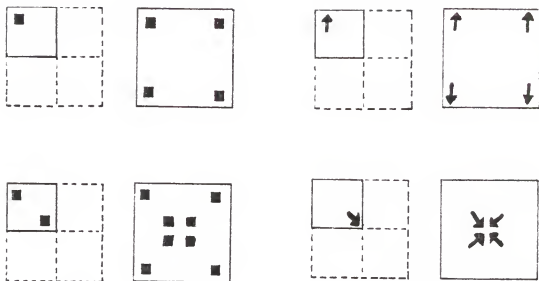


Figure D.2
Paper Folding Designs

Students were given verbal clues during the activities to remind them how to unfold the paper.

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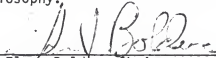
BIOGRAPHICAL SKETCH

Marian Tillotson was born on May 7, 1949 in Washington, D.C., and lived in its suburb, Silver Spring, Maryland, until 1960. Marian then moved with her family to Europe and lived in Frankfurt, Germany, for two years. Following a year back in Silver Spring, Maryland, Marian and her family spent two years on Okinawa, a territory of Japan. After one more year back in Silver Spring, Maryland, Marian moved to Miami, Florida, where she graduated from high school.

Marian attended the University of Miami in Coral Gables, Florida, for four years. There she earned the Bachelor of Education and Bachelor of Arts degrees in mathematics and German. Marian remained in Miami two additional years teaching mathematics at a local high school.

Marian then moved to Gainesville, Florida to begin graduate school. She earned a Master of Arts in Teaching degree at the University of Florida but remained in Gainesville, Florida to continue her studies. Marian moved to Valdosta, Georgia, when she was offered the opportunity to become a mathematics consultant for twelve school systems at the Coastal Plains Cooperative Educational Service Agency, part of a state sponsored system designed to provide educational expertise to all Georgia school systems regardless of size. She remained with that agency until moving to Covington, Georgia, to become the Mathematics and Testing Coordinator for the Newton County school system. Marian completed her Doctor of Philosophy degree while working in Covington.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.


Elroy Bolduc, Chairman
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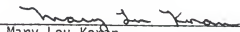
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This dissertation was submitted to the Graduate Faculty of the Division of Curriculum and Instruction in the College of Education and to the Graduate School, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1984

Dean for Graduate Studies and Research